Quiz 10, Math 246, Professor David Levermore
Thursday, 3 December 2020

This formative assessment helps you see how well you understand the material. To get an accurate assessment please do not use books, notes, or electronic aids. Show your reasoning for full credit. Good luck!

(1) [3] Consider the system $\mathbf{x}' = \mathbf{C}\mathbf{x}$ where $\mathbf{C} = \begin{pmatrix} 5 & -3 \\ 6 & -1 \end{pmatrix}$.

(a) [1] Classify its phase-plane portrait.
(b) [1] Determine the stability of the origin for this system.
(c) [1] Sketch its phase-plane portrait.

(2) [3] Consider the system $\mathbf{x}' = \mathbf{B}\mathbf{x}$ where the 2×2 matrix $\mathbf{B}$ has eigenpairs

$$\left(-2\,,\,\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right)\,, \qquad \left(-1\,,\,\begin{pmatrix} 1 \\ -2 \end{pmatrix}\right)\,.$$

(a) [1] Classify its phase-plane portrait.
(b) [1] Determine the stability of the origin for this system.
(c) [1] Sketch its phase-plane portrait.

(3) [4] Consider the system

$$u' = -4u + 2v - 9v^2\,, \qquad v' = u + 4v\,.$$

(a) [2] This system has two stationary points. Find them.
(b) [2] Find a nonconstant function $H(u, v)$ such that every orbit of this system satisfies $H(u, v) = c$ for some constant c. (No sketch is needed.)