Quiz 9, Math 246, Professor David Levermore
Thursday, 12 November 2020

This formative assessment helps you see how well you understand the material. To get an accurate assessment please do not use books, notes, or electronic aids. Show your reasoning for full credit. Good luck!

(1) [2] Recast the equation $y'''' - \sin(y+t)y'' + e^{y'}y = 0$ as a first-order system of ordinary differential equations.

(2) [3] Determine the interval of definition for the solution of the initial-value problem

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{t^2 - 16} \begin{pmatrix} e^{2t} & \cos(2t) \\ t^2 & e^{-3t} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \log(t^2) \\ 0 \end{pmatrix}, \qquad \begin{pmatrix} x(-1) \\ y(-1) \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \end{pmatrix}.$$

Give your reasoning.

(3) [5] Consider the vector-valued functions $\mathbf{x}_1(t) = \begin{pmatrix} 1 \\ 2t^2 \end{pmatrix}$, $\mathbf{x}_2(t) = \begin{pmatrix} -t^4 \\ 2 - t^6 \end{pmatrix}$.

(a) [2] Compute their Wronskian $\mathrm{Wr}[\mathbf{x}_1, \mathbf{x}_2](t)$.
(b) [3] Find $\mathbf{A}(t)$ such that $\mathbf{x}_1$, $\mathbf{x}_2$ is a fundamental set of solutions to $\mathbf{x}' = \mathbf{A}(t)\mathbf{x}$.