

**Quiz 1 Solutions, Math 246, Professor David Levermore**  
**Thursday, 3 September 2020**

(1) [4] For each of the following ordinary differential equations, determine its order and whether it is linear or nonlinear. If it is nonlinear, write down term that makes it so.

(a)  $v''' + e^v v' = 2 \cos(3t)v + 4e^{5t}$

(b)  $u'''' + 3 \sin(2t)u' = 2 \cos(4t)$

**Solution (a).** The equation is *third order* because  $v'''$  is the highest order derivative that appears. It is *nonlinear* because of the  $e^v v'$  term.

**Solution (b).** The equation is *fourth order* because  $u''''$  is the highest order derivative that appears. It is *linear* and is already in normal form.

(2) [2] What is the interval of definition for the solution of the initial-value problem

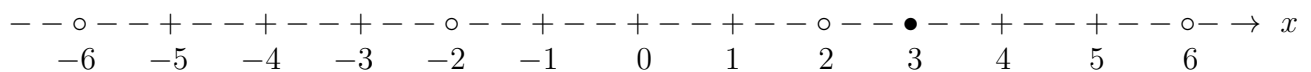
$$\frac{dw}{dx} + \frac{e^x}{x^2 - 4} w = \frac{\sin(x)}{x^2 - 36}, \quad w(3) = -7.$$

(You do not need to solve the differential equation, but you must give your reasoning!)

**Solution.** This is a nonhomogeneous linear equation that is already in normal form. Observe that

- its coefficient  $\frac{e^x}{x^2 - 4}$  is undefined at  $x = \pm 2$  and is continuous elsewhere;
- its forcing  $\frac{\sin(x)}{x^2 - 36}$  is undefined at  $x = \pm 6$  and is continuous elsewhere.

The initial time is  $x = 3$ . The situation is pictured on the  $x$ -axis as follows.



Therefore *the interval of definition for the solution is (2, 6)* because:

- *the initial time  $x = 3$  is in (2, 6),*
- *the coefficient and forcing are both continuous over (2, 6),*
- *the coefficient is undefined at  $x = 2$ ,*
- *the forcing is undefined at  $x = 6$ .*

**Remark.** All four reasons had to be given for full credit. The first two reasons are why a unique solution exists over (2, 6), The third reason is why the left endpoint is  $x = 2$ . The fourth reason is why the right endpoint  $x = 6$ .

(3) [4] Solve the initial-value problem

$$(1+t) \frac{dy}{dt} + 2y = 6t^2, \quad y(0) = 5.$$

**Solution.** This is a nonhomogeneous linear equation. Its normal form is

$$\frac{dy}{dt} + \frac{2}{1+t}y = \frac{6t^2}{1+t}.$$

The coefficient and forcing are both undefined at  $t = -1$  and continuous elsewhere. The initial time is  $t = 0$ . The interval of definition of the solution is thereby  $(-1, \infty)$ .

An integrating factor is  $e^{A(t)}$  where  $A'(t) = 2/(1+t)$ . Setting

$$A(t) = 2 \log(|1+t|) = \log((1+t)^2),$$

we obtain  $e^{A(t)} = (1+t)^2$ . The equation thereby has the integrating factor form

$$\frac{d}{dt}((1+t)^2y) = (1+t)^2 \cdot \frac{6t^2}{1+t} = 6t^2 + 6t^3.$$

Integrating both sides yields

$$(1+t)^2y = 2t^3 + \frac{3}{2}t^4 + c.$$

Imposing the initial condition  $y(0) = 5$  gives

$$(1+0)^2 5 = 2 \cdot 0^3 + \frac{3}{2} \cdot 0^4 + c,$$

whereby  $c = 5$ . Therefore the solution of the initial-value problem is

$$y = \frac{2t^3 + \frac{3}{2}t^4 + 5}{(1+t)^2}.$$

**Remark.** The interval of definition of this solution is indeed  $(-1, \infty)$ .