Math 246, Professor David Levermore
Group Work Exercises for Discussion
Wednesday, 9 December 2020

Set A of Group Work Exercises [3]

Consider the planar system

$$\dot{p} = 2p + q \,, \qquad \dot{q} = 5p - 2q + 3p^2 \,.$$

Last week we saw that its stationary points are $(0,0)$ and $(-3,6)$. We also saw that this system is Hamiltonian. Use teamwork for each exercise.

A.1. Compute the Jacobian matrix of this system and evaluate the coefficient matrix for its linearization at each stationary point. Check that each stationary point is regular — i.e. that the determinant of its coefficient matrix is nonzero.

A.2. Use linearization to classify the type and stability of each stationary point. Sketch a single phase-plane portrait for the system in the pq-plane that shows its behavior near each stationary point. Carefully mark all sketched orbits with arrows! (Different teammates should do each stationary point.)

A.3. Last week we saw that this system has the Hamiltonian

$$H(p,q) = \tfrac{1}{2}q^2 + 2pq - \tfrac{5}{2}p^2 - p^3 \,.$$

Find q as a function of p for each level set $H(p,q) = c$ that contains a stationary point. Use these functions to add to the sketch of the phase-plane portrait all orbits on each level set $H(p,q) = c$ that contains a stationary point. Carefully mark all sketched orbits with arrows! (You may use a computer.)

Set B of Group Work Exercises [3]

Consider the planar system

$$\dot{u} = u^2 + v - 9 \,, \qquad \dot{v} = -2uv \,.$$

Last week we saw that its stationary points are $(-3,0)$, $(3,0)$, and $(0,9)$. We also saw that this system is Hamiltonian. Use teamwork.

B.1. Compute the Jacobian matrix of this system and evaluate the coefficient matrix for its linearization at each stationary point. Check that each stationary point is regular — i.e. that the determinant of its coefficient matrix is nonzero.

B.2. Use linearization to classify the type and stability of each stationary point. Sketch a single phase-plane portrait for the system in the uv-plane that shows its behavior near each stationary point. Carefully mark all sketched orbits with arrows! (Different teammates should do each stationary point.)

B.3. Last week we saw that this system has the Hamiltonian

$$H(u,v) = u^2v + \tfrac{1}{2}v^2 - 9v \,.$$

Add to the sketch of the phase-plane portrait all orbits on each level set $H(u,v) = c$ that contains a saddle point. Carefully mark all sketched orbits with arrows! (Do not use a computer.)

Set C of Group Work Exercises [4]

Consider the planar system

$$x' = (21 - 3x - 6y)x\,, \qquad y' = (5 - 2x - y)y\,.$$

Use teamwork. The classification of each stationary point can be done independently.

C.1. This system has four stationary points. Find them.

C.2. Compute the Jacobian matrix of this system and evaluate the coefficient matrix for its linearization at each stationary point. Check that each stationary point is regular — i.e. that the determinant of its coefficient matrix is nonzero.

C.3. Use linearization classify the type and stability of each stationary point. Sketch a single phase-plane portrait for the system in the xy-plane that shows its behavior near each stationary point. Carefully mark all sketched orbits with arrows! (Different teammates should do each stationary point.)

C.4. Add any semistationary orbits to the sketch of the phase-plane portrait. Carefully mark all sketched orbits with arrows! Explain why this system is not conservative (i.e. why it does not have an integral) over the entire xy-plane.