Math 246, Professor David Levermore
Group Work Exercises for Discussion
Wednesday, 2 December 2020

Set A of Group Work Exercises [4]

Consider each of the following linear planar system.

(a) Classify its phase-plane portrait.
(b) Determine whether the stability of the origin is attracting, repelling, stable but not attracting, or unstable but not repelling.
(c) Sketch its phase-plane portrait. Carefully mark all sketched orbits with arrows.

These problems are independent. Work as a team!

A.1. $\mathbf{x}' = \mathbf{A}\mathbf{x}$ where $\mathbf{A} = \begin{pmatrix} 6 & -1 \\ 3 & 2 \end{pmatrix}$.

A.2. $\mathbf{x}' = \mathbf{B}\mathbf{x}$ where $\mathbf{B} = \begin{pmatrix} -6 & -1 \\ 4 & -2 \end{pmatrix}$.

A.3. $\mathbf{x}' = \mathbf{C}\mathbf{x}$ where $\mathbf{C} = \begin{pmatrix} -2 & 2 \\ 3 & 3 \end{pmatrix}$.

A.4. $\mathbf{x}' = \mathbf{K}\mathbf{x}$ where $\mathbf{K} = \begin{pmatrix} -5 & 5 \\ -5 & 1 \end{pmatrix}$.

Set B of Group Work Exercises [3]

Sketch a phase-plane portrait for the system

$$\dot{p} = 2p + q\,, \qquad \dot{q} = 5p - 2q + 3p^2\,.$$

Carefully mark all sketched orbits with arrows! The first two problems are independent.

B.1. Plot all stationary points and sketch any semistationary orbits of this system.

B.2. Find a nonconstant function $H(p, q)$ such that every orbit of this system satisfies $H(p, q) = c$ for some constant c.

B.3. Classify each stationary point. Sketch all orbits on the level set $H(p, q) = c$ for each value of c that corresponds to a stationary point that is a saddle.

Set C of Group Work Exercises [3]

Sketch a phase-plane portrait for the system

$$\dot{u} = u^2 + v - 9\,, \qquad \dot{v} = -2uv$$

Carefully mark all sketched orbits with arrows! The first two problems are independent.

C.1. Plot all stationary points and sketch any semistationary orbits of this system.

C.2. Find a nonconstant function $H(u, v)$ such that every orbit of this system satisfies $H(u, v) = c$ for some constant c.

C.3. Classify each stationary point. Sketch all orbits on the level set $H(u, v) = c$ for each value of c that corresponds to a stationary point that is a saddle.