Math 246, Professor David Levermore
Group Work Exercises for Discussion
Wednesday, 18 November 2020

Set A of Group Work Exercises [4]

Find a general solution for each of the following systems.

A.1. $\mathbf{x}' = \mathbf{A}\mathbf{x}$ where $\mathbf{A} = \begin{pmatrix} 7 & -1 \\ -5 & 3 \end{pmatrix}$.

A.2. $\mathbf{y}' = \mathbf{B}\mathbf{y}$ where $\mathbf{B} = \begin{pmatrix} 7 & -2 \\ 2 & 3 \end{pmatrix}$.

A.3. $\mathbf{z}' = \mathbf{C}\mathbf{z}$ where $\mathbf{C} = \begin{pmatrix} 7 & -2 \\ 4 & 3 \end{pmatrix}$.

A.4. $\mathbf{x}' = \mathbf{A}\mathbf{x}$ where $\mathbf{A} = \begin{pmatrix} 0 & -8 & 1 \\ 8 & 0 & 4 \\ -1 & -4 & 0 \end{pmatrix}$.

Set B of Group Work Exercises [3]

Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 7 & -2 \\ 4 & 3 \end{pmatrix} .$$

B.1. Find all the eigenvalues of $\mathbf{A}$.

B.2. For each eigenvalue of $\mathbf{A}$ find all of its eigenvectors.

B.3. Diagonalize $\mathbf{A}$.

Set C of Group Work Exercises [3]

A 4×4 matrix $\mathbf{B}$ has the eigenpairs

$$\left(-3\,, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right) , \qquad \left(-1\,, \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \right) , \qquad \left(2\,, \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \right) , \qquad \left(4\,, \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} \right) .$$

C.1. Give an invertible matrix $\mathbf{V}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{B} = \mathbf{V}\mathbf{D}\mathbf{V}^{-1}$. (You do not have to compute $\mathbf{V}^{-1}$.

C.2. Give a fundamental matrix for the system $\mathbf{x}' = \mathbf{B}\mathbf{x}$.

C.3. Compute $\det(\mathbf{B})$. Hint: Use the fact that $\mathbf{B} = \mathbf{V}\mathbf{D}\mathbf{V}^{-1}$ along with the fact from Chapter 3 that for any square matrices $\mathbf{A}$ and $\mathbf{C}$ we have $\det(\mathbf{A}\mathbf{C}) = \det(\mathbf{A})\det(\mathbf{C})$.