Math 246, Professor David Levermore
Group Work Exercises for Discussion
Wednesday, 4 November 2020

Set A of Group Work Exercises [3]

Use the table on page 2.

A.1. Use the Laplace transform to solve the initial-value problem

$$y''' + 25y' = 21\cos(2t)\,, \qquad y(0) = 0\,, \quad y'(0) = 0\,, \quad y''(0) = 0\,.$$

A.2. Use the Laplace transform to compute the Green function $g(t)$ for the operator

$$\mathrm{L} = \mathrm{D}^3 + 25\mathrm{D}\,, \qquad \text{where} \quad \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}\,.$$

A.3. Compute the natural fundamental set of solutions $N_0(t)$, $N_1(t)$, $N_2(t)$ associated with initial time 0 for the operator $\mathrm{L} = \mathrm{D}^3 + 25\mathrm{D}$.

Set B of Group Work Exercises [3]

Use the table on page 2.

B.1. Use the Laplace transform to solve the initial-value problem

$$v'''' + 18v'' + 81v = 50\sin(2t)\,, \qquad v(0) = 0\,, \quad v'(0) = 0\,, \quad v''(0) = 0\,, \quad v'''(0) = 0\,.$$

B.2. Use the Laplace transform to compute the Green function $g(t)$ for the operator

$$\mathrm{L} = \mathrm{D}^4 + 18\mathrm{D}^2 + 81\,, \qquad \text{where} \quad \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}\,.$$

B.3. Compute the natural fundamental set of solutions $N_0(t)$, $N_1(t)$, $N_2(t)$, $N_3(t)$ associated with initial time 0 for the operator $\mathrm{L} = \mathrm{D}^4 + 18\mathrm{D}^2 + 36$.

Set C of Group Work Exercises [4]

Use the table on page 2. Consider the initial-value problem

$$x'' + 25x = f(t)\,, \qquad x(0) = 2\,, \quad x'(0) = -4\,.$$

where

$$f(t) = \begin{cases} t & \text{for } 0 \le t < 4\,, \\ 8 - t & \text{for } 4 \le t < 8\,, \\ 0 & \text{for } 8 \le t\,. \end{cases}$$

C.1. Compute $F(s) = \mathcal{L}[f](s)$.

C.2. Compute $X(s) = \mathcal{L}[x](s)$.

C.3. Compute $x(t)$.

C.4. Solve the initial-value problem with $f(t)$ replaced by $12\,\delta(t-6)$.

Table of Laplace Transforms

$$\mathcal{L}[t^n e^{at}](s) = \frac{n!}{(s-a)^{n+1}} \qquad \text{for } s > a\,.$$

$$\mathcal{L}[e^{at}\cos(bt)](s) = \frac{s-a}{(s-a)^2+b^2} \qquad \text{for } s > a\,.$$

$$\mathcal{L}[e^{at}\sin(bt)](s) = \frac{b}{(s-a)^2+b^2} \qquad \text{for } s > a\,.$$

$$\mathcal{L}[t^n j(t)](s) = (-1)^n J^{(n)}(s) \qquad \text{where } J(s) = \mathcal{L}[j(t)](s)\,.$$

$$\mathcal{L}[e^{at} j(t)](s) = J(s-a) \qquad \text{where } J(s) = \mathcal{L}[j(t)](s)\,.$$

$$\mathcal{L}[u(t-c)j(t-c)](s) = e^{-cs}J(s) \qquad \text{where } J(s) = \mathcal{L}[j(t)](s),\ c \geq 0,$$

and u is the unit step function.

$$\mathcal{L}[\delta(t-c)j(t)](s) = e^{-cs}j(c) \qquad \text{where } c \geq 0$$

and δ is the unit impulse.