Math 246, Professor David Levermore
Group Work Exercises for Discussion
Wednesday, 28 October 2020

Set A of Group Work Exercises [3]

Compute the following Laplace transforms from the definition. Identify the values of s where each is defined. Here $u(t)$ is the unit step function.

A.1. $\mathcal{L}[f](s)$ where $f(t) = u(t-4)t^2 e^{3t}$.

A.2. $\mathcal{L}[g](s)$ where $g(t) = u(t-3)t\cos(4t)$.

A.3. $\mathcal{L}[h](s)$ where $h(t) = u(t-5)e^{2t}\sin(3t)$.

Set B of Group Work Exercises [4]

Refer to the table on page 2 if needed. Here $u(t)$ s the unit step function.

B.1. Find the exponential order of $f(t) = u(t-3)e^{-5t}\cos(4t)$.

B.2. Find $F(s) = \mathcal{L}[f](s)$ where $f(t) = u(t-3)e^{-5t}\cos(4t)$.

B.3. Find the exponential order of $h(t) = u(t-3)t\, e^{-5t}\cos(4t)$.

B.4. Find $H(s) = \mathcal{L}[h](s)$ where $h(t) = u(t-3)t\, e^{-5t}\cos(4t)$.

Set C of Group Work Exercises [3]

Refer to the table on page 2 if needed.

C.1. Compute $X(s) = \mathcal{L}[x](s)$ where $x(t)$ is the solution of the initial-value problem

$$x''' + 25x' = 24\sin(3t)\,,$$
$$x(0) = 0\,, \quad x'(0) = 0\,, \quad x''(0) = 0\,.$$

C.2. Compute $Y(s) = \mathcal{L}[y](s)$ where $y(t)$ is the solution of the initial-value problem

$$y'''' + 12y'' + 36y = 36\cos(3t)\,,$$
$$y(0) = 3\,, \quad y'(0) = -2\,, \quad y''(0) = -5\,, \quad y'''(0) = 4\,.$$

C.3. Compute $Z(s) = \mathcal{L}[z](s)$ where $z(t)$ is the solution of the initial-value problem

$$z'''' + 12z'' + 36z = u(t-3)e^{-t}\,,$$
$$z(0) = 1\,, \quad z'(0) = 0\,, \quad z''(0) = 0\,, \quad z'''(0) = 2\,.$$

Here $u(t)$ is the unit step function.

Table of Laplace Transforms

$$\mathcal{L}[t^n e^{at}](s) = \frac{n!}{(s-a)^{n+1}} \qquad \text{for } s > a\,.$$

$$\mathcal{L}[e^{at}\cos(bt)](s) = \frac{s-a}{(s-a)^2+b^2} \qquad \text{for } s > a\,.$$

$$\mathcal{L}[e^{at}\sin(bt)](s) = \frac{b}{(s-a)^2+b^2} \qquad \text{for } s > a\,.$$

$$\mathcal{L}[t^n j(t)](s) = (-1)^n J^{(n)}(s) \qquad \text{where } J(s) = \mathcal{L}[j(t)](s)\,.$$

$$\mathcal{L}[e^{at} j(t)](s) = J(s-a) \qquad \text{where } J(s) = \mathcal{L}[j(t)](s)\,.$$

$$\mathcal{L}[u(t-c)j(t-c)](s) = e^{-cs}J(s) \qquad \text{where } J(s) = \mathcal{L}[j(t)](s),\ c \geq 0,$$

and u is the unit step function.