Math 246, Professor David Levermore
Group Work Exercises for Discussion
Wednesday, 21 October 2020

Set A of Group Work Exercises [3]

When a 10 gram mass is hung vertically from a spring, at rest it stretches the spring 9.8 cm. (Gravitational acceleration is $g = 980$ cm/sec^2.) A damper imparts a force of 500 dynes (1 dyne = 1 gram cm/sec^2) when the speed of the mass is 2 cm/sec. Assume that the spring force is proportional to displacement, that the damping force is proportional to velocity, and that there are no other forces. At $t = 0$ the mass is displaced 2 cm above its rest position and is released with a downward velocity of 4 cm/sec.

A.1. Give the natural frequency and natrual period of the spring. (Give your reasoning!)

A.2. Is this system undamped, under damped, critically damped, or over damped? (Give your reasoning!)

A.3. Give an initial-value problem that governs the displacement $h(t)$ for $t > 0$. (DO NOT solve this initial-value problem, just write it down!)

Set B of Group Work Exercises [3]

The displacement $h(t)$ of a spring-mass system is governed by the equation

$$\ddot{h} + 2\eta\dot{h} + 289h = 255\cos(\omega t) - 136\sin(\omega t)\,,$$

where $\eta \geq 0$ is the damping rate and $\omega > 0$ is the forcing frequency.

B.1. Determine the values of η for which the system is:
  (a) undamped,
  (b) under damped,
  (c) critically damped,
  (d) over damped.

B.2. Give the damped frequency and damped period of the system when $\eta = 8$.

B.3. Give the forcing and the steady-state solution in phasor form when $\eta = 8$.

Set C of Group Work Exercises [4]

Consider the nonhomogeneous equation

$$(1+t)t\,q'' - (1+3t)q' + 3q = \frac{24t^2}{1+3t} \qquad \text{over } t > 0\,.$$

C.1. Show that $1 + 3t$ and $(1 + t)^3$ are a fundamental set of solutions for the associated homogeneous equation.

C.2. Set up and solve the linear alegbraic system for $u_1'(t)$ and $u_2'(t)$ from the variation of parameters method. (You do not have to integrate here.)

C.3. Give the Green function $G(t, s)$ for this equation and set up the Green formula for the particular solution that satisfies the initial conditions $q(3) = q'(3) = 0$. (You do not have to integrate here.)

C.4. Give a general solution of the equation. (You have to integrate here.)