Math 246, Professor David Levermore
Group Work Exercises for Discussion
Wednesday, 7 October 2020

Set A of Group Work Exercises [3]

Consider the differential equation

$$u'' + 6u' + 9u = 0\,.$$

It has solutions $u = e^{-3t}$ and $u = t\,e^{-3t}$.

A.1. Compute the Wronskian $\mathrm{Wr}[e^{-3t}, t\,e^{-3t}](t)$. How can we know that $\mathrm{Wr}[e^{-3t}, t\,e^{-3t}](t)$ is proportional to e^{-6t} before we compute it? (Hint: Abel.)

A.2. Why are e^{-3t} and $t\,e^{-3t}$ linearly independent functions?

A.3. Find the natural fundamental set of solutions to the equation associated with $t = 0$.

Set B of Group Work Exercises [3]

Consider the differential equation

$$(t^2 + 4)v'' - 2tv' + 2v = 0\,.$$

It has solution $v = t$.

B.1. Find a general solution of its associated Abel equation.

B.2. Find a general solution to it.

B.3. Find the natural fundamental set of solutions to it associated with $t = 2$.

Set C of Group Work Exercises [4]

C.1. Give a real general solution to the equation

$$u'''' - 5u''' - 6u'' = 0\,.$$

C.2. Give a real general solution to the equation

$$\big(\mathrm{D}^2 + 8\mathrm{D} + 41\big)^3(\mathrm{D} + 7)^2 w = 0\,.$$

C.3. Give a real general solution to the equation

$$y'' - 36y = 8e^{2t}\,,$$

given that $-\frac{1}{4}e^{2t}$ is a solution to it.

C.4. Give a real general solution to the equation

$$v'''' + 13v'' + 36v = 10e^{t}\,,$$

given that $\frac{1}{5}e^{t}$ is a solution to it.