Math 246, Professor David Levermore
Group Work Exercises for Discussion
Wednesday, 30 September 2020

First Set of Group Work Exercises [4]

Consider the fourth-order, linear differential operator

$$\mathrm{L}(t) = \mathrm{D}^4 - \frac{2t}{4+t^2}\,\mathrm{D}^3 - \frac{\sin(2t)}{36-t^2}\,\mathrm{D}^2 + \frac{e^t}{9-t}\,\mathrm{D} + \frac{16+t^4}{\sin(t)}\,.$$

(1) Evaluate $\mathrm{L}(t)e^{3t}$.

(2) Give the interval of definition for the solution of the initial-value problem

$$\mathrm{L}(t)y = \frac{e^{2t}}{8+t}\,, \qquad y'''(7) = y''(7) = y'(7) = y(7) = 0\,.$$

(3) Give the interval of definition for the solution of the initial-value problem

$$\mathrm{L}(t)y = \frac{e^{2t}}{8+t}\,, \qquad y'''(-7) = y''(-7) = y'(-7) = y(-7) = 5\,.$$

(4) Suppose that $Y_1(t)$, $Y_2(t)$, $Y_3(t)$, and $Y_4(t)$ solve $\mathrm{L}(t)y = 0$ and that their Wronskian satisfies $\mathrm{Wr}[Y_1, Y_2, Y_3, Y_4](1) = 10$. Compute $\mathrm{Wr}[Y_1, Y_2, Y_3, Y_4](t)$. (Hint: Abel)

Second Set of Group Work Exercises [3]

Let $U_1(t) = (1+3t)$ and $U_2(t) = (1+t)^3$.

(1) Verify that $U_1(t)$ and $U_2(t)$ solve

$$t(1+t)\,u'' - (1+3t)\,u' + 3u = 0\,.$$

(2) Compute the Wronskian $\mathrm{Wr}[U_1, U_2](t)$. (Evaluate the determinant and simplify.)

(3) Solve the general initial-value problem

$$t(1+t)\,u'' - (1+3t)\,u' + 3u = 0\,, \qquad u(1) = u_0\,, \quad u'(1) = u_1\,.$$

Give the interval of definition for this solution.

Third Set of Group Work Exercises [3]

Let $V_1(t) = e^{2t}$, $V_2(t) = t\,e^{2t}$, $V_3(t) = t^2 e^{2t}$.

(1) Verify that $V_1(t)$, $V_2(t)$ and $V_3(t)$ solve

$$v''' - 6v'' + 12v' - 8v = 0\,.$$

(2) Compute the Wronskian $\mathrm{Wr}[V_1, V_2, V_3](t)$. (Evaluate the determinant and simplify.) Verify that it satisfies $w' - 6w = 0$.

(3) Solve the general initial-value problem

$$v''' - 6v'' + 12v' - 8v = 0\,, \qquad v(0) = v_0\,, \quad v'(0) = v_1\,, \quad v''(0) = v_2\,.$$