Math 246, Professor David Levermore
Group Work Exercises for Discussion
Wednesday, 23 September 2020

First Set of Group Work Exercises [3]

Let $v(t)$ be the solution of the initial-value problem

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 4v - v^2\,, \qquad v(0) = 3\,.$$

Your answers may be left as arithmetic expressions.

(1) Approximate $v(.2)$ using two steps of the explicit Euler method.

(2) Approximate $v(.2)$ using one step of the Runge-trapezoidal method.

(3) The differential equation is autonomous. Use a phase-line portrait to describe how the solution $v(t)$ of the initial-value problem behaves.
- Is $v(t)$ an increasing or decreasing function of t?
- How does $v(t)$ behave as $t \to \infty$?

Are the numerical approximations consistent with this information?

Second Set of Group Work Exercises [3]

Suppose we have used a numerical method to approximate the solution of an initial-value problem over the time interval $[5, 15]$ with 1000 uniform time steps.

(1) About how many uniform time steps are needed to reduce the global error of our approximation by a factor of $\frac{1}{4096}$ if the method we had used was each of the following? (Notice that $8^4 = 4096$.)
(a) explicit Euler method
(b) Runge-Kutta method
(c) Runge-trapeziodal method
(d) Runge-midpoint method

(2) For each method in the previous exercise give the associated step size as a fraction.

(3) Suppose we had used a sixth order method.
(a) How many uniform time steps are needed to reduce the global error of our approximation by a factor of $\frac{1}{4096}$?
(b) Give the associated step size as a fraction.

Third Set of Group Work Exercises [4]

Determine if each of the following differential forms is exact. If it is then find an implicit general solution. Otherwise find an integrating factor. (You do not need to find a general solution in the last case.) (Distribute the effort among your group members.)

(1) $\big(e^x + \cos(x+y)\big)\,\mathrm{d}x + \big(\cos(y) + \cos(x+y)\big)\,\mathrm{d}y = 0\,.$

(2) $\sin(y)\,\mathrm{d}x + \cos(y)\,\mathrm{d}y = 0\,.$

(3) $y\cos(x)\,\mathrm{d}x + \big(3\sin(x) + 5y^2\big)\,\mathrm{d}y = 0\,.$

(4) $(ye^{2xy} + 2x^3)\,\mathrm{d}x + (xe^{2xy} + e^y)\,\mathrm{d}y = 0\,.$