Math 246, Professor David Levermore
Group Work Exercises for Discussion
Wednesday, 16 September 2020

First Set of Group Work Exercises [3]

A tank with a capacity of 55 liters initially contains 19 liters of brine (salt water) with a salt concentration of 4 grams per liter (gr/lit). At time $t = 0$ brine with a salt concentration of 7 grams per liter (gr/lit) begins to flow into the tank at a constant rate of 3 liters per minute (lit/min) and the well-stirred mixture flows out of the tank at a constant rate of 1 liters per minute (lit/min).

(1) Find $V(t)$, the volume of brine in the tank as a function of time for $t > 0$. Give the time at which the tank overflows.

(2) Write down an initial-value problem that governs $S(t)$, the amount of salt in the tank for $t > 0$ until the tank overflows. (Do not solve the initial-value problem!)

(3) Give the interval of definition for the solution of the initial-value problem.

Second Set of Group Work Exercises [3]

In the absence of predators the population of mosquitoes in a certain area would increase at a rate proportional to its current population such that it would triple every five weeks. There are 180,000 mosquitoes in the area when a flock of birds arrives that eats 40,000 mosquitoes per week.

(1) Write down an initial-value problem that governs $M(t)$, the population of mosquitoes in the area after the flock of birds arrives.

(2) Is the flock of birds large enough to control the mosquitoes?

(3) How do the answers to the previous two questions change if there were 200,000 mosquitoes in the area when the same flock of birds arrives?

Third Set of Group Work Exercises [4]

Consider the initial-value problem

$$\ddot{r} = -\frac{2a^2}{r^5}, \qquad r(0) = r_o, \quad \dot{r}(0) = v_o,$$

where $a > 0$, $r_o > 0$, and $v_o > 0$.

(1) Write down the auxiliary initial-value problem.

(2) Find the reduced autonomous initial-value problem.

(3) Find a condition that a, r_o, and v_o satisfy if and only if the reduced autonomous equation has a stationary solution.

(4) When the reduced autonomous equation has a stationary solution, find its stationary point and sketch its phase-line portrait for $r > 0$.