Math 246 Exam 2
Professor David Levermore
Thursday, 22 October 2020
due by 4:00pm Friday, 23 October

- This is an "open-book exam". You may refer to the on-line text or the solutions to any of our past quizzes or group work.
- **Work on your own!** Do not consult with anyone or with any interactive resource.
- **On the top of the first page of your answers please hand copy and sign the University Honor Pledge below.**

  University Honor Pledge: *I pledge on my honor that I have not given or received any unauthorized assistance on this examination.*
- Answers should be handwritten. Indicate where the answer to each part of each problem is located. Cross out work that you do not want considered.
- **Your reasoning must be given for full credit!**
- Uploaded your answers to ELMS as a pdf file by **4:00pm on Friday, October 23.**

(1) [4] Give the interval of definition for the solution of the initial-value problem

$$y''' + \frac{e^{3t}}{\sin(2t)}\,y'' + \frac{2+t}{8-t}\,y = \frac{\cos(4t)}{9-t^2}\,, \qquad y(-7) = y'(-7) = y''(-7) = 5\,.$$

(2) [12] The functions e^{7t} and e^{-7t} are a fundamental set of solutions to $v'' - 49v = 0$.
  (a) [8] Solve the general initial-value problem

$$v'' - 49v = 0\,, \qquad v(0) = v_0\,, \quad v'(0) = v_1\,.$$

  (b) [4] Find the associated natural fundamental set of solutions to $v'' - 49v = 0$.

(3) [4] Suppose that $Z_1(t)$, $Z_2(t)$, $Z_3(t)$, and $Z_4(t)$ solve the differential equation

$$z'''' + 5z''' + e^{3t}z'' + \sin(5t)z' + t^2 z = 0\,,$$

Suppose we know that $\mathrm{Wr}[Z_1, Z_2, Z_3, Z_4](3) = 4$. Find $\mathrm{Wr}[Z_1, Z_2, Z_3, Z_4](t)$.

(4) [12] Let L be a linear ordinary differential operator with constant coefficients. Suppose that all the roots of its characteristic polynomial (listed with their multiplicities) are $-2+i4$, $-2+i4$, $-2+i4$, $-2-i4$, $-2-i4$, $-2-i4$, -3, -3, 0, 0, 0.
  (a) [1] Give the order of L. (Give your reasoning!)
  (b) [6] Give a real general solution of the homogeneous equation $\mathrm{L}u = 0$.
  (c) [5] Write down the form for the particular solution needed to start the Undetermined Coefficients method for the equation $\mathrm{L}v = t^2 e^{-2t}\cos(4t)$.

(5) [8] Find a real general solution of the equation $y'''' + 12y'' + 36y = 36\cos(3t)$.

(6) [8] What answer will be produced by the following Matlab commands?

```
>> syms x(t)
>> ode = diff(x,t,2) + 3*diff(x,t) − 10*x == 28*t*exp(2*t);
>> xSol(t) = dsolve(ode)
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(7) [8] Compute the Green function $g(t)$ associated with the differential operator

$$\mathrm{D}^2 + 6\mathrm{D} + 45\,, \qquad \text{where} \quad \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}\,.$$

(8) [8] Solve the initial-value problem

$$h'' + 6h' + 45h = \frac{72e^{-3t}}{\sin(6t)}\,, \qquad h(\tfrac{\pi}{4}) = h'(\tfrac{\pi}{4}) = 0\,.$$

(9) [10] Consider the nonhomogeneous initial-value problem

$$t\,p'' - (1+2t)p' + 2p = \frac{24t^2}{1+2t}\,, \qquad p(2) = p'(2) = 0\,.$$

(a) [3] Show that $1+2t$ and e^{2t} are a fundamental set of solutions for the associated homogeneous equation.

(b) [7] Solve the nonhomogeneous initial-value problem.

(10) [8] Give a real general solution of the equation

$$\mathrm{D}^2 v - 5\mathrm{D}v - 36v = 12\cos(2t) - 5\sin(2t)\,, \qquad \text{where} \quad \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}\,.$$

(11) [8] The vertical displacement of a spring-mass system is governed by the equation

$$\ddot{h} + 40\dot{h} + 481h = a\cos(\omega t - \phi)\,,$$

where $a > 0$, $\omega > 0$, and $0 \leq \phi < 2\pi$. Assume CGS units.

(a) [2] Give the natural frequency and period of the system.

(b) [2] Show the system is under damped and give its damping rate.

(c) [4] Give the steady state solution in its phasor form $\mathrm{Re}(\Gamma\, e^{i\omega t})$.

(12) [10] When a 10 gram mass is hung vertically from a spring, at rest it stretches the spring 20 cm. (Gravitational acceleration is $g = 980$ cm/sec^2.) A damper imparts a damping force of 560 dynes (1 dyne = 1 gram cm/sec^2) when the speed of the mass is 4 cm/sec. Assume that the spring force is proportional to displacement, that the damping force is proportional to velocity, and that there are no other forces. At $t = 0$ the mass is displaced 3 cm below its rest position and is released with an upward velocity of 2 cm/sec.

(a) [6] Give an initial-value problem that governs the displacement $h(t)$ for $t > 0$. (DO NOT solve this initial-value problem, just write it down!)

(b) [2] Is this system undamped, under damped, critically damped, or over damped? (Give your reasoning!)

(c) [2] Find the damped frequency and damped period of the system. (Give your reasoning!)