MATH 416 Homework 8
Due Wednesday, 8 April 2020

1. Let $\{x_j\}_{j=0}^n \subset \mathbb{R}$ be symmetric about 0. This means if $x \in \{x_j\}_{j=0}^n$ then $-x \in \{x_j\}_{j=0}^n$.
   a. Show that if $f : \mathbb{R} \to \mathbb{R}$ is even (i.e. $f(-x) = f(x)$) that then its polynomial interpolant of degree at most n through the nodes $\{x_j\}_{j=0}^n$ is also even.
   b. Show that if $f : \mathbb{R} \to \mathbb{R}$ is odd (i.e. $f(-x) = -f(x)$) that then its polynomial interpolant of degree at most n through the nodes $\{x_j\}_{j=0}^n$ is also odd.

   Hint: Use the uniqueness of the interpolant.

   Remark. These facts can simplify interpolation because an even polynomial $p(x)$ has only even powers of x, while an odd polynomial $p(x)$ has only odd powers of x.

2. Find the six Lagrange interpolating polynomials for the nodes $\{-5, -3, -1, 1, 3, 5\}$. On a single graph plot these polynomials over the interval $[-6, 6]$.

3. Generate the Chebyshev polynomials $\{T_n(x)\}_{n=0}^6$ and find their roots. (Show work!) On a single graph plot the polynomials $\{T_n(x)\}_{n=1}^6$ over the interval $[-1, 1]$.

4. Compute the polynomials of degree at most 5 that interpolate the values of the function $f(x) = 1/(1 + x^2)$ at
   a. the uniform nodes $\{-5, -3, -1, 1, 3, 5\}$;
   b. the Chebyshev nodes $\{6r : T_6(r) = 0\}$.

   Plot these two interpolants over the interval $[-6, 6]$. Which gives a better approximation to $f(x)$ over $[-6, 6]$?

5. Plot the continuous, piecewise linear approximation of $f(x) = 1/(1 + x^2)$ over $[-6, 6]$ that is linear over the subintervals $[-6, -4]$, $[-4, -2]$, $[-2, 0]$, $[0, 2]$, $[2, 4]$, and $[4, 6]$. How does this piecewise interpolation compare with the two found in the previous exercise?