MATH 416 Homework 3
Due Wednesday, 1 April 2020

For every $n \in \mathbb{Z}_+$ let P^n denote all polynomials with real coefficients of degree at most n. We saw that the set of monomials $\{t^k\}_{k=0}^n$ is a basis for P^n. Therefore P^n has dimension $n+1$.

1. Consider the mapping $S : P^n \to P^n$ given by
$$S(p)(t) = p'(t) + 2p(t)\,, \qquad \text{for every } p \in P^n\,.$$
Give the matrix representation on this mapping with respect to the basis $\{t^k\}_{k=0}^n$ for $n = 5$.

2. Give the null space and range of the mapping S from the previous problem when $n = 5$.

3. Consider the mapping $T : P^n \to P^n$ given by
$$T(p)(t) = t\,p'(t) - 3p(t)\,, \qquad \text{for every } p \in P^n\,.$$
Give the matrix representation on this mapping with respect to the basis $\{t^k\}_{k=0}^n$ for $n = 5$.

4. Give the null space and range of the mapping T from the previous problem when $n = 5$.

5. Equip P^n with the inner product
$$\langle p\,,\, q\rangle = \frac{1}{2}\int_{-1}^{1} p(t)\,q(t)\,t^2 \mathrm{d}t\,.$$
Give the adjoints S^* and T^* with respect to this inner product of the mappings S and T from the above problems when $n = 5$.