# MATH 416 Homework 2
## Due Wednesday, 1 April 2020

Let $\mathbf{A} \in \mathbb{R}^{3\times 4}$ be given by

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & -1 & 2 & 1 \\ 2 & 3 & 0 & 1 \end{pmatrix}$$

Define the linear map $T : \mathbb{R}^4 \to \mathbb{R}^3$ by $T(\mathbf{x}) = \mathbf{A}\mathbf{x}$ for every $\mathbf{x} \in \mathbb{R}^4$. The null space and range of $T$ are defined respectively by

$$\begin{aligned} \mathrm{Null}(T) &= \left\{ \mathbf{x} \in \mathbb{R}^4 \,:\, T(\mathbf{x}) = \mathbf{0} \right\}, \\ \mathrm{Range}(T) &= \left\{ T(\mathbf{x}) \in \mathbb{R}^3 \,:\, \mathbf{x} \in \mathbb{R}^4 \right\}. \end{aligned}$$

1. Compute $\mathrm{Null}(T)$ and give its dimension.

2. Compute $\mathrm{Range}(T)$ and give its dimension.

3. Equip $\mathbb{R}^4$ with the usual Euclidean inner product. Give the orthogonal projection of $\mathbb{R}^4$ onto $\mathrm{Null}(T)$.

4. Equip $\mathbb{R}^3$ with the inner product defined by

   $$\langle \mathbf{x}\,,\, \mathbf{y} \rangle = \mathbf{x}^{\mathrm{T}}\mathbf{G}\mathbf{y} \qquad \text{for every } \mathbf{x}, \mathbf{y} \in \mathbb{R}^3\,,$$

   where $\mathbf{G}$ is the diagonal matrix

   $$\mathbf{G} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

   Give the orthogonal projection of $\mathbb{R}^3$ onto $\mathrm{Range}(T)$.

5. Equip $\mathbb{R}^4$ and $\mathbb{R}^3$ with the inner products given in the previous two problems. Compute $T^* : \mathbb{R}^3 \to \mathbb{R}^4$, the adjoint of $T$ with respect to these inner products.