

**Quiz 10, Math 246, Professor David Levermore**  
**Tuesday, 26 November 2019**

**Your Name:**

**Discussion Instructor (circle one):**      Sam Potter      Nathan Yu      David Russell  
**Discussion Time (circle one):**      9:00      11:00      12:00

**No books, notes, calculators, or any electronic devices.**  
**Show your reasoning for full credit. Good luck!**

- (1) [3] Consider the system  $\mathbf{x}' = \mathbf{B}\mathbf{x}$  where  $\mathbf{B} = \begin{pmatrix} -4 & 6 \\ -3 & 2 \end{pmatrix}$ .
- (a) [1] Classify its phase-plane portrait.
  - (b) [1] Determine the stability of the origin for this system.
  - (c) [1] Sketch its phase-plane portrait.

- (2) [3] Consider the system  $\mathbf{x}' = \mathbf{C}\mathbf{x}$  where the  $2 \times 2$  matrix  $\mathbf{C}$  has eigenpairs

$$\left(1, \begin{pmatrix} 1 \\ 3 \end{pmatrix}\right), \quad \left(2, \begin{pmatrix} 3 \\ -1 \end{pmatrix}\right).$$

- (a) [1] Classify its phase-plane portrait.
- (b) [1] Determine the stability of the origin for this system.
- (c) [1] Sketch its phase-plane portrait.

- (3) [4] Consider the system

$$x' = 3x - y, \quad y' = 5x - 3y + 2x^2.$$

- (a) [2] This system has two stationary points. Find them.
- (b) [2] Find a nonconstant function  $H(x, y)$  such that every orbit of this system satisfies  $H(x, y) = c$  for some constant  $c$ .