

**Quiz 9, Math 246, Professor David Levermore**  
**Tuesday, 12 November 2019**

**Your Name:**

**Discussion Instructor (circle one):**      Sam Potter      Nathan Yu      David Russell  
**Discussion Time (circle one):**      9:00      11:00      12:00

**No books, notes, calculators, or any electronic devices.**  
**Show your reasoning for full credit. Good luck!**

- (1) [5] Consider the vector-valued functions  $\mathbf{x}_1(t) = \begin{pmatrix} 1 \\ -t^2 \end{pmatrix}$ ,  $\mathbf{x}_2(t) = \begin{pmatrix} 2t^4 \\ 4 - t^6 \end{pmatrix}$ .
- (a) [2] Compute their Wronskian  $\text{Wr}[\mathbf{x}_1, \mathbf{x}_2](t)$ .
- (b) [3] Find  $\mathbf{A}(t)$  such that  $\mathbf{x}_1, \mathbf{x}_2$  is a fundamental set of solutions to  $\mathbf{x}' = \mathbf{A}(t)\mathbf{x}$ .

- (2) [4] Let  $\mathbf{B} = \begin{pmatrix} -1 & -2 \\ 4 & -5 \end{pmatrix}$ . Compute  $e^{t\mathbf{B}}$ .

- (3) [1] Suppose that  $e^{t\mathbf{C}} = e^{3t} \begin{pmatrix} \cosh(4t) & -\frac{1}{2} \sinh(4t) \\ -2 \sinh(4t) & \cosh(4t) \end{pmatrix}$ .
- Compute the Green matrix  $\mathbf{G}(t, s)$  for the system  $\mathbf{x}' = \mathbf{C}\mathbf{x} + \mathbf{f}(t)$ .