Math 246, Professor David Levermore
Group Work Exercises for Discussion 15
Monday, 9 December 2019

Answers to the following exercises should be worked out on the board space for your group. **Your reasoning must be shown for full credit!**

First Set of Group Work Exercises [2]

The following systems are from Problems 1 and 2 of Quiz 11. You can use information from the Quiz 11 Solutions.

(1) Consider the system from Problem 1 of Quiz 11,

$$x' = -4x + y\,, \qquad y' = 5x - 5x^2\,.$$

Sketch a single phase-plane portrait for the system in the xy-plane that shows its behavior near each stationary point. Carefully mark all sketched orbits with arrows!

(2) Consider the system from Problem 2 of Quiz 11,

$$u' = -2u + v\,, \qquad v' = -3v + 3u^2\,.$$

Sketch a single phase-plane portrait for the system in the uv-plane that shows its behavior near each stationary point. Carefully mark all sketched orbits with arrows!

Remark. These systems do not have semistationary solutions, are not Hamiltonian, and do not have a separable orbit equation.

Second Set of Group Work Exercises [5]

Consider the system

$$\dot{p} = (10 - 5p + 3q)p\,, \qquad \dot{q} = (10 + 3p - 5q)q\,.$$

(1) Identify this system as being either a predator-prey, competing species, or cooperating species model. Explain your answer.

(2) Find the stationary points of this system. Compute the Jacobian matrix of this system and evaluate the coefficient matrix for its linearization at each stationary point. Check that each stationary point is regular and classify its type and stability.

(3) Sketch a single phase-plane portrait for the system in the pq-plane that shows its behavior near each stationary point. Carefully mark all sketched orbits with arrows! Explain why this system is not conservative (i.e. why it does not have an integral) over the first quadrant of the pq-plane.

(4) Add any semistationary orbits to the sketch of the phase-plane portrait. Carefully mark all sketched orbits with arrows! Explain why orbits in the first quadrant do not leave the first quadrant.

(5) Show that there are solutions of this system that satisfy $q = p$ and

$$\dot{p} = (10 - 2p)p\,.$$

Add the orbits of these solutions to the sketch of the phase-plane portrait. Carefully mark all sketched orbits with arrows!

Third Set of Group Work Exercises [3]

Consider the system

$$\dot{p} = (10 - 5p + 3q)p\,, \qquad \dot{q} = (10 + 3p - 5q)q\,.$$

The following exercises review some Exam 1 material. They can be done independently.

(1) Its semistationary solutions with $q = 0$ stationary satisfy

$$\dot{p} = (10 - 5p)p\,.$$

Find an explicit general solution of this equation.

(2) Its semistationary solutions with $q = 0$ stationary satisfy

$$\dot{p} = (10 - 5p)p\,.$$

In a single tp-plane sketch a graph of the three solutions of this equation that satisfy the initial conditions $p(0) = -1$, $p(0) = 1$, and $p(0) = 3$.

(3) It solutions with $q = p$ satisfy

$$\dot{p} = (10 - 2p)p\,.$$

Find an explicit general solution of this equation.