Math 246, Professor David Levermore
Group Work Exercises for Discussion 13
Monday, 25 November 2019

Answers to the following exercises should be worked out on the board space for your group.
Your reasoning must be shown for full credit!

First Set of Group Work Exercises [3]

For each of the following matrices $\mathbf{A}$, consider the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

(a) Classify its phase-plane portrait.
(b) Determine the stability of the origin for this system.
(c) Sketch its phase-plane portrait. Carefully mark all sketched orbits with arrows!

The problems are independent. Work as a team!

(1) $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$

(2) $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ -1 & 2 \end{pmatrix}$

(3) $\mathbf{A} = \begin{pmatrix} -4 & -1 \\ 2 & -2 \end{pmatrix}$

Second Set of Group Work Exercises [3]

Sketch a phase-plane portrait for the system

$$\dot{p} = 2p + q\,, \qquad \dot{q} = 5p - 2q + 3p^2\,.$$

Carefully mark all sketched orbits with arrows! The first two problems are independent.

(1) Plot all stationary points of this system.
(2) Find a nonconstant function $H(p, q)$ such that every orbit of this system satisfies $H(p, q) = c$ for some constant c.
(3) Classify each stationary point. Sketch all orbits on the level set $H(p, q) = c$ for each value of c that corresponds to a stationary point that is a saddle.

Third Set of Group Work Exercises [4]

Sketch a phase-plane portrait for the system

$$\dot{u} = u^2 + v - 9\,, \qquad \dot{v} = -2uv$$

Carefully mark all sketched orbits with arrows! The first three problems are independent.

(1) Plot all stationary points of this system.
(2) Plot all semistationary orbits of this system.
(3) Find a nonconstant function $H(u, v)$ such that every orbit of this system satisfies $H(u, v) = c$ for some constant c.
(4) Classify each stationary point. Sketch all orbits on the level set $H(u, v) = c$ for each value of c that corresponds to a stationary point that is a saddle.