Math 246, Professor David Levermore
Group Work Exercises for Discussion 12
Monday, 18 November 2019

Answers to the following exercises should be worked out on the board space for your group. **Your reasoning must be shown for full credit!**

First Set of Group Work Exercises [5]

Considered the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 2 \\ 2 & 2 & 0 \end{pmatrix} .$$

Work as a team. Problem 4 can be worked independently of Problems 1 through 3. Problem 3 will take the most time.

(1) Compute the characteristic polynomial $p_{\mathbf{A}}(z)$ of $\mathbf{A}$.

(2) Find the roots of $p_{\mathbf{A}}(z)$.

(3) Compute the natural fundamental set of solutions associated with the initial-time 0 for the operator $p_{\mathbf{A}}(\mathrm{D})$.

(4) Compute $\mathbf{A}^2$.

(5) Compute $e^{t\mathbf{A}}$.

Second Set of Group Work Exercises [5]

Again considered the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 2 \\ 2 & 2 & 0 \end{pmatrix} .$$

In Problem 1 you can use the answer to Problem 2 from the First Set of Group Work Exercises. Work as a team. In Problem 2 the three eigenvectors can be found independently. In Problem 4 the entries in the matrix product $\mathbf{V}^{\mathrm{T}}\mathbf{V}$ can be found independently.

(1) Find all of the eigenvalues of $\mathbf{A}$.

(2) Find an eigenvector for each eigenvalue of $\mathbf{A}$.

(3) Give an invertible matrix $\mathbf{V}$ and a diagonal matrix $\mathbf{D}$ such that $e^{t\mathbf{A}} = \mathbf{V}e^{t\mathbf{D}}\mathbf{V}^{-1}$. (Here you do not have to compute either $\mathbf{V}^{-1}$ or $e^{t\mathbf{A}}$!)

(4) Compute $\mathbf{V}^{\mathrm{T}}$ and $\mathbf{V}^{\mathrm{T}}\mathbf{V}$.

(5) Find $\mathbf{V}^{-1}$. (Hint: Use the result of Problem 4.)

Remark. The nice structure that should emerge in your answers to Problems 4 and 5 are a consequence of the fact that $\mathbf{A}$ is symmetric. Similar structure arises for any real matrix that is symmetric ($\mathbf{A}^{\mathrm{T}} = \mathbf{A}$), skew-symmetric ($\mathbf{A}^{\mathrm{T}} = -\mathbf{A}$), or normal ($\mathbf{A}^{\mathrm{T}}\mathbf{A} = \mathbf{A}\mathbf{A}^{\mathrm{T}}$).