

Math 246, Professor David Levermore
Group Work Exercises for Discussion 11
Monday, 11 November 2019

Answers to the following exercises should be worked out on the board space for your group.
Your reasoning must be shown for full credit!

First Set of Group Work Exercises [3]

Consider the vector-valued functions $\mathbf{x}_1(t) = \begin{pmatrix} 1 \\ e^{3t} \end{pmatrix}$, $\mathbf{x}_2(t) = \begin{pmatrix} t^2 \\ e^{3t} \end{pmatrix}$.

- (1) Compute their Wronskian $\text{Wr}[\mathbf{x}_1, \mathbf{x}_2](t)$.
- (2) Find $\mathbf{A}(t)$ such that $\mathbf{x}_1, \mathbf{x}_2$ is a fundamental set of solutions to $\mathbf{x}' = \mathbf{A}(t)\mathbf{x}$.
- (3) Give the natural fundamental matrix for $t_I = 2$ of the system $\mathbf{x}' = \mathbf{A}(t)\mathbf{x}$.

Second Set of Group Work Exercises [4]

Let $\mathbf{C} = \begin{pmatrix} 1 & -2 \\ 5 & 3 \end{pmatrix}$.

- (1) Give a general solution to the system $\mathbf{x}' = \mathbf{C}\mathbf{x}$.
- (2) Solve the initial-value problem $\mathbf{x}' = \mathbf{C}\mathbf{x}$, $\mathbf{x}(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.
- (3) Give the natural fundamental matrix for $t_I = 5$ of the system $\mathbf{x}' = \mathbf{C}\mathbf{x}$.
- (4) Give an eigenpair for each eigenvalue of \mathbf{C} .

Third Set of Group Work Exercises [3]

Suppose that $e^{t\mathbf{B}} = e^{2t} \begin{pmatrix} \cosh(3t) & \frac{1}{5} \sinh(3t) \\ 5 \sinh(3t) & \cosh(3t) \end{pmatrix}$.

- (1) Compute the Green matrix $\mathbf{G}(t, s)$ for the system $\mathbf{x}' = \mathbf{B}\mathbf{x} + \mathbf{f}(t)$.
- (2) Solve the initial-value problem $\mathbf{y}' = \mathbf{B}\mathbf{y}$, $\mathbf{y}(0) = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$.
- (3) Find \mathbf{B} .