Math 246, Professor David Levermore
Group Work Exercises for Discussion 9
Monday, 28 October 2019

Answers to the following exercises should be worked out on the board space for your group. **Your reasoning must be shown for full credit!**

First Set of Group Work Exercises [4]

Use the table on page 2 if needed.

(1) Find the exponential order of $f(t) = u(t-3)e^{-5t}\cos(4t)$.

(2) Find $F(s) = \mathcal{L}[f](s)$ where $f(t) = u(t-3)e^{-5t}\cos(4t)$.

(3) Find the exponential order of $h(t) = u(t-3)t^2e^{-5t}\cos(4t)$.

(4) Find $H(s) = \mathcal{L}[h](s)$ where $h(t) = u(t-3)t^2e^{-5t}\cos(4t)$.

Second Set of Group Work Exercises [3]

Use the table on page 2 if needed.

(1) Use the Laplace transform to solve the initial-value problem

$$v''' + 16v' = 24\cos(2t)\,, \qquad v(0) = 0\,, \quad v'(0) = 0\,, \quad v''(0) = 0\,.$$

(2) Use the Laplace transform to compute the Green function $g(t)$ for the operator

$$\mathrm{L} = \mathrm{D}^3 + 16\mathrm{D}\,, \qquad \text{where} \quad \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}\,.$$

(3) Compute the natural fundamental set of solutions $N_0(t)$, $N_1(t)$, $N_2(t)$ associated with initial time 0 for the operator $\mathrm{L} = \mathrm{D}^3 + 16\mathrm{D}$.

Third Set of Group Work Exercises [3]

Use the table on page 2 if needed.

(1) Find $x(t) = \mathcal{L}^{-1}[X](t)$ where $X(s) = e^{-2s}\dfrac{2s+11}{s^2-10s+21}$.

(2) Find $y(t) = \mathcal{L}^{-1}[Y](t)$ where $Y(s) = e^{-4s}\dfrac{3s+21}{s^2+10s+29}$.

(3) Find $z(t) = \mathcal{L}^{-1}[Z](t)$ where $Z(s) = e^{-3s}\dfrac{26}{(s^2-9)(s^2+4)}$.

Warning. On Quiz 7 you will have to compute $\mathcal{L}[f](s)$ for some $f(t)$ using the definition of the Laplace transform.

Table of Laplace Transforms

$$\mathcal{L}[t^n e^{at}](s) = \frac{n!}{(s-a)^{n+1}} \qquad \text{for } s > a\,.$$

$$\mathcal{L}[e^{at}\cos(bt)](s) = \frac{s-a}{(s-a)^2+b^2} \qquad \text{for } s > a\,.$$

$$\mathcal{L}[e^{at}\sin(bt)](s) = \frac{b}{(s-a)^2+b^2} \qquad \text{for } s > a\,.$$

$$\mathcal{L}[t^n j(t)](s) = (-1)^n J^{(n)}(s) \qquad \text{where } J(s) = \mathcal{L}[j(t)](s)\,.$$

$$\mathcal{L}[e^{at} j(t)](s) = J(s-a) \qquad \text{where } J(s) = \mathcal{L}[j(t)](s)\,.$$

$$\mathcal{L}[u(t-c)j(t-c)](s) = e^{-cs}J(s) \qquad \text{where } J(s) = \mathcal{L}[j(t)](s),\ c \geq 0,$$
and u is the unit step function.