**Math 246, Professor David Levermore**
**Group Work Exercises for Discussion 7**
**Monday, 14 October 2019**

Answers to the following exercises should be worked out on the board space for your group.
**Your reasoning must be shown for full credit!**

## First Set of Group Work Exercises [2]

Consider the differential operator

$$\mathrm{L} = (D^2 + 4\mathrm{D} + 13)^2(\mathrm{D} + 4)^3\mathrm{D}^2\,.$$

(You saw this operator last week.)

(1) Identify the degree, characteristic and mulitplicity of the forcing for the equation

$$\mathrm{L}v = t^4 e^{-2t}\sin(3t)\,.$$

(2) Give the undetermined coefficient form of the particular solution for the equation

$$\mathrm{L}v = t^4 e^{-2t}\sin(3t)\,.$$

(Do not carry out the method, just give the form.)

## Second Set of Group Work Exercises [4]

Consider the differential operator

$$\mathrm{L} = D^2 + 8\mathrm{D} + 16\,.$$

(1) Give a real general solution of $\mathrm{L}y = 0$.

(2) Compute the Green function of L.

(3) Find a particular solution of

$$\mathrm{L}u = \frac{8e^{-4t}}{3+t}\,.$$

(4) Solve the initial-value problem

$$\mathrm{L}u = \frac{8e^{-4t}}{1+4t^2}\,, \qquad u(0) = 0\,, \quad u'(0) = 5\,.$$

**Third Set of Group Work Exercises** [4]

The displacement $h(t)$ of a spring-mass system is governed by the equation

$$\ddot{h} + 2\eta\dot{h} + 169h = 169\sin(\omega t)\,,$$

where $\eta \geq 0$ is the damping rate and $\omega > 0$ is the forcing frequency.

(1) Give the natural frequency and natrual period of the spring.

(2) Determine the values of $\eta$ for which the system is:
   (a) undamped,
   (b) under damped,
   (c) critically damped,
   (d) over damped.

(3) Give the damped frequency and damped period of the system when $\eta = 5$.

(4) Give the forcing and the steady-state solution in phasor form when $\eta = 5$.