Math 246, Professor David Levermore
Group Work Exercises for Discussion 5
Monday, 30 September 2019

Answers to the following exercises should be worked out on the board space for your group.
Your reasoning must be shown for full credit!

First Set of Group Work Exercises [4]

Consider the fourth-order, linear differential operator

$$\mathrm{L}(t) = \mathrm{D}^4 + \frac{e^{3t}}{1+t^2}\,\mathrm{D}^3 - \frac{4+t^4}{\sin(t)}\,\mathrm{D}^2 + \frac{\cos(2t)}{25-t^2}\,\mathrm{D} + \frac{1}{8-t}\,.$$

(1) Evaluate $\mathrm{L}(t)e^{2t}$.

(2) Give the interval of definition for the solution of the initial-value problem

$$\mathrm{L}(t)y = \frac{e^{2t}}{9+t}\,, \qquad y'''(7) = y''(7) = y'(7) = y(7) = 0\,.$$

(3) Give the interval of definition for the solution of the initial-value problem

$$\mathrm{L}(t)y = \frac{e^{2t}}{9+t}\,, \qquad y'''(-7) = y''(-7) = y'(-7) = y(-7) = 5\,.$$

(4) Suppose that $Y_1(t)$, $Y_2(t)$, $Y_3(t)$, and $Y_4(t)$ solve $\mathrm{L}(t)y = 0$ and that their Wronskian satisfies $\mathrm{Wr}[Y_1, Y_2, Y_3, Y_4](1) = 2$. Compute $\mathrm{Wr}[Y_1, Y_2, Y_3, Y_4](t)$. (Hint: Abel)

Second Set of Group Work Exercises [3]

The functions $V_1(t) = e^{-3t}$ and $V_2(t) = t\,e^{-3t}$ are linearly independent solutions of

$$v'' + 6v' + 9v = 0\,.$$

(1) Solve the general initial-value problem

$$v'' + 6v' + 9v = 0\,, \qquad v(0) = v_0\,, \quad v'(0) = v_1\,.$$

(2) Find the natural fundamental set of solutions associated with the initial time $t = 0$ for the equation

$$v'' + 6v' + 9v = 0\,.$$

(3) Solve the initial-value problems

$$v'' + 6v' + 9v = 0\,, \qquad v(0) = 1\,, \quad v'(0) = 0\,;$$
$$v'' + 6v' + 9v = 0\,, \qquad v(0) = 0\,, \quad v'(0) = 1\,.$$

Third Set of Group Work Exercises [3]

The functions $U_1(t) = (1 + 2t)$ and $U_2(t) = (1 + t)^2$ solve

$$t(1 + t)\, u'' - (1 + 2t)\, u' + 2u = 0\,.$$

(1) Compute the Wronskian $\mathrm{Wr}[U_1, U_2](t)$. (Evaluate the determinant and simplify.)

(2) Solve the general initial-value problem

$$t(1 + t)\, u'' - (1 + 2t)\, u' + 2u = 0\,, \qquad u(1) = u_0\,, \quad u'(1) = u_1\,.$$

Give the interval of definition for this solution.

(3) Find the natural fundamental set of solutions associated with the initial time $t = 1$ for the equation

$$t(1 + t)\, u'' - (1 + 2t)\, u' + 2u = 0\,.$$