Math 246, Professor David Levermore
Group Work Exercises for Discussion 4
Monday, 23 September 2019

Answers to the following exercises should be worked out on the board space for your group.
Your reasoning must be shown for full credit!

First Set of Group Work Exercises [6]

Problem 5 on Exam 1 concerned the phase-line portrait of

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{(u^2-4)(u+6)^2}{(u^2+4)(u-6)} \,.$$

Refer to the Exam 1 Solutions and answer the following.

(1) For each stationary point identify the set of initial values $u(0)$ such that the solution $u(t)$ converges to that stationary point as $t \to \infty$.

(2) Give all initial values $u(0)$ for which $u(t)$ has interval of definition $(-\infty, \infty)$.

(3) Give all initial values $u(0)$ for which $u(t)$ has interval of definition (t_*, ∞) for some finite time t_*. Identify if $u(t)$ or $\dot{u}(t)$ blows up as t approaches t_* from above.

(4) Give all initial values $u(0)$ for which $u(t)$ has interval of definition such that $(-\infty, t_*)$ for some finite time t_*. Identify if $u(t)$ or $\dot{u}(t)$ blows up as t approaches t_* from below.

(5) Sketch a graph of u versus t showing several solution curves. The graph should show all of the stationary solutions as well as solution curves above and below each of them. Every value of u for which the equation is defined should lie on at least one sketched solution curve.

(6) Give the partial fraction identity that is needed to find an implicit solution of this equation analytically. (You do not have to integraate to find an implicit solution.)

Second Set of Group Work Exercises [4]

Problem 1 on Exam 1 concerned the initial-value problem

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 3t^2(2x - x^2)\,, \qquad x(0) = 1\,.$$

Here we consider the more general initial-value problem

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 3t^2(2x - x^2)\,, \qquad x(0) = x_o\,.$$

Refer to the Exam 1 Solutions and answer the following.

(1) Find the solution $x(t)$ of the more general initial-value problem. The answer depends upon x_o as well as t. (You can use steps that appear in the Exam 1 Solutions.)

(2) Identify all initial values x_o for which the interval of definition of $x(t)$ is $(-\infty, \infty)$.

(3) For all other initial values of x_o give the interval of of definition of $x(t)$.

(4) Suppose that $x_o > 0$. Evaluate $\lim_{t\to\infty} x(t)$.