Math 246, Professor David Levermore
Group Work Exercises for Discussion 3
Monday, 16 September 2019

First Set of Group Work Exercises [4]

Problem 1 on Quiz 2 concerned the phase-line portrait of

$$\frac{\mathrm{d}w}{\mathrm{d}t} = \frac{(w+4)(w+1)^3(w-5)^2}{(1+w^2)^2(w-2)} \,.$$

The solution of Problem 2 on Quiz 2 was the initial-value problem

$$\frac{\mathrm{d}S}{\mathrm{d}t} = 20 - \frac{2}{13+2t}\, S \,, \qquad S(0) = 39 \,.$$

Refer to the Quiz 2 solutions and answer the following.

(1) For each stationary point identify the set of initial values $w(0)$ such that the solution $w(t)$ converges to that stationary point as $t \to \infty$.

(2) Identify all initial values $w(0)$ such that the interval of definition of the solution $w(t)$ is $(-\infty, \infty)$.

(3) Sketch a graph of w versus t showing several solution curves. The graph should show all of the stationary solutions as well as solution curves above and below each of them. Every value of w for which the equation is defined should lie on at least one sketched solution curve.

(4) Give the interval of definition for the solution $S(t)$ of the initial-value problem found for Problem 2 of the quiz. It should be restricted to times that make sense for the problem.

Second Set of Group Work Exercises [3]

Consider the initial-value problem

$$\ddot{x} = -2x + 2x^3 \,, \qquad x(0) = 0 \,, \quad \dot{x}(0) = v_o \,.$$

(1) Write down and solve the associated auxiliary equation. The answer depends on v_o.

(2) Assume that $v_o > 0$. Write down the associated reduced autonomous equation. Determine the values of v_o for which it has stationary solutions. When it has them, find them.

(3) Find $x(t)$ when $v_o = 1$.

Third Set of Group Work Exercises [3]

Let $u(t)$ be the solution of the initial-value problem

$$\frac{\mathrm{d}u}{\mathrm{d}t} = 2u - u^2\,, \qquad u(0) = 3\,.$$

(1) Approximate $u(.2)$ using two steps of the explicit Euler method.

(2) Approximate $u(.2)$ using one step of the Runge-midpoint method.

(3) The differential equation is autonomous. Use a phase-line portrait to describe how the solution $u(t)$ of the initial-value problem behaves.
- Is $u(t)$ an increasing or decreasing function of t?
- How does $u(t)$ behave as $t \to \infty$?

Are the numerical approximations consistent with this information?