Math 246, Professor David Levermore
Group Work Exercises for Discussion 2
Monday, 9 September 2019

First Set of Group Work Exercises [4]

Problem 3 on Quiz 1 concerned the solution $y(x)$ of the initial-value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = e^x \, \frac{y^2 - 9}{2y} \,, \qquad y(0) = -5 \,.$$

Refer to the Quiz 1 Soluions and answer the following.

(1) How does $y(x)$ behave as $x \to -\infty$?

(2) How does $y(x)$ behave as $x \to \infty$?

(3) What is the explicit solution of the differential equation that satisfies the initial condition $y(0) = 1$?

(4) What is the interval of definition for the solution that satisfies the initial condition $y(0) = 1$?

Second Set of Group Work Exercises [3]

Consider the equation

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{(u+5)^2(u+1)^3(7-u)}{u-3} \,.$$

Let $u_1(t)$ and $u_2(t)$ be the solutions of it that satisfy $u_1(2) = -3$ and $u_2(-1) = 5$. (You do not need to find these solutions!)

(1) Sketch the phase-line portrait for the equation.

(2) Classify each stationary point as being either stable, unstable, or semistable.

(3) Evaluate $\lim_{t\to\infty} \big(u_2(t) - u_1(t)\big)$.

Third Set of Group Work Exercises [3]

In the absence of predators the population of mosquitoes in a certain area would increase at a rate proportional to its current population such that it would triple every five weeks. There are 180,000 mosquitoes in the area when a flock of birds arrives that eats 40,000 mosquitoes per week.

(1) Write down an initial-value problem that governs $M(t)$, the population of mosquitoes in the area after the flock of birds arrives.

(2) Is the flock of birds large enough to control the mosquitoes?

(3) How do the answers to the previous two questions change if there were 200,000 mosquitoes in the area when the same flock of birds arrives?

Remark. In the above exercises it is helpful to know that $\frac{12}{11} < \log(3) < \frac{11}{10}$.