Second In-Class Exam
Math 246, Professor David Levermore
Thursday, 17 October 2019

Your Name:____

UMD SID:____

Discussion Instructor (circle one): Sam Potter Nathan Yu David Russell
Discussion Time (circle one): 9:00 11:00 12:00

No books, notes, calculators, or any electronic devices. If you need more space to answer a problem then use the back of one of these pages. **Do not separate the pages!** Indicate where the answer to each part of each problem is located. Cross out work that you do not want considered. **Your reasoning must be given for full credit!** Good luck!

Please copy and sign the University Honor Pledge below.
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University Honor Pledge: *I pledge on my honor that I have not given or received any unauthorized assistance on this examination.* ____

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Signature:____

Problem 1:	________/4	Problem 2:	________/12
Problem 3:	________/4	Problem 4:	________/12
Problem 5:	________/8	Problem 6:	________/8
Problem 7:	________/8	Problem 8:	________/8
Problem 9:	________/10	Problem 10:	________/8
Problem 11:	________/10	Problem 12:	________/8
		Total Score:	________/100
		Grade:	________

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(1) [4] Give the interval of definition for the solution of the initial-value problem

$$x''' - \frac{\cos(3t)}{t^2 - 16}\, x' + \frac{e^t}{\sin(t)}\, x = \frac{1}{1 + t^2}\,, \qquad x(5) = x'(5) = x''(5) = -2\,.$$

(2) [12] The functions t and t^2 are a fundamental set of solutions to $t^2 y'' - 2ty' + 2y = 0$ over $t > 0$.

(a) [8] Solve the general initial-value problem

$$t^2 y'' - 2ty' + 2y = 0\,, \qquad y(1) = y_0\,, \quad y'(1) = y_1\,.$$

(b) [4] Find the associated natural fundamental set of solutions to $t^2 y'' - 2ty' + 2y = 0$.

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(3) [4] Suppose that $Z_1(t)$, $Z_2(t)$, $Z_3(t)$, and $Z_4(t)$ solve the differential equation

$$z'''' + 3z''' + \sin(2t)z' + e^t z' + 6z = 0\,,$$

Suppose we know that $\mathrm{Wr}[Z_1, Z_2, Z_3, Z_4](0) = 5$. Find $\mathrm{Wr}[Z_1, Z_2, Z_3, Z_4](t)$.

(4) [12] Let L be a linear ordinary differential operator with constant coefficients. Suppose that all the roots of its characteristic polynomial (listed with their multiplicities) are $-3 + i2$, $-3 + i2$, $-3 - i2$, $-3 - i2$, -5, -5, -5, 0, 0.

(a) [2] Give the order of L.

(b) [7] Give a real general solution of the homogeneous equation $\mathrm{L}u = 0$.

(c) [3] Give the degree d, characteristic $\mu + i\nu$, and multiplicity m for the forcing of the nonhomogeneous equation $\mathrm{L}v = t^4 e^{-3t} \sin(2t)$.

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(5) [8] What answer will be produced by the following MATLAB commands?

```
>> syms w(t)
>> ode = diff(w,t,2) − diff(w,t) − 12*w == 12*exp(3*t);
>> wSol(t) = dsolve(ode)
```

(6) [8] Find a particular solution $q_P(t)$ of the equation $q'' - 4q = 8t\, e^{2t}$.

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(7) [8] Compute the Green function $g(t)$ associated with the differential operator

$$\mathrm{D}^2 + 6\mathrm{D} + 9\,, \qquad \text{where} \quad \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}\,.$$

(8) [8] Solve the initial-value problem

$$v'' + 6v' + 9v = \frac{9e^{-3t}}{1+t}\,, \qquad v(0) = v'(0) = 0\,.$$

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(9) [10] The functions $1 + 2t$ and t^2 are solutions of the homogeneous equation

$$(1+t)t\,x'' - (1+2t)x' + 2x = 0 \qquad \text{over } t > 0\,.$$

(You do not have to check that this is true!)

(a) [3] Show that these functions are linearly independent.

(b) [7] Give a general solution of the nonhomogeneous equation

$$(1+t)t\,y'' - (1+2t)y' + 2y = \frac{8t\,(1+t)^2}{1+2t} \qquad \text{over } t > 0\,.$$

(10) [8] Give a real general solution of the equation

$$\mathrm{D}^2 u - 4\mathrm{D}u + 20u = 4\cos(2t) - 3\sin(2t)\,, \qquad \text{where} \quad \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}\,.$$

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(11) [10] The vertical displacement of a spring-mass system is governed by the equation

$$\ddot{h} + 10\dot{h} + 169h = \alpha\cos(\omega t) + \beta\sin(\omega t)\,,$$

where $\alpha \neq 0$, $\beta \neq 0$, and $\omega > 0$. Assume CGS units.

(a) [2] Give the natural frequency and period of the system.

(b) [4] Show the system is under damped and give its damped frequency and period.

(c) [4] Give the steady state solution in its phasor form $\mathrm{Re}(\Gamma\, e^{i\omega t})$.

(12) [8] When a 10 gram mass is hung vertically from a spring, at rest it stretches the spring 20 cm. (Gravitational acceleration is $g = 980$ cm/sec^2.) A dashpot imparts a damping force of 420 dynes (1 dyne = 1 gram cm/sec^2) when the speed of the mass is 3 cm/sec. Assume that the spring force is proportional to displacement, that the damping force is proportional to velocity, and that there are no other forces. At $t = 0$ the mass is displaced 5 cm above its rest position and is released with a downward velocity of 2 cm/sec.

(a) [6] Give an initial-value problem that governs the displacement $h(t)$ for $t > 0$. (DO NOT solve this initial-value problem, just write it down!)

(b) [2] Is this system undamped, under damped, critically damped, or over damped? (Give your reasoning!)