First In-Class Exam
Math 246, Professor David Levermore
Thursday, 19 September 2019

Your Name:____

UMD SID:____

Discussion Instructor (circle one): Sam Potter Nathan Yu David Russell
Discussion Time (circle one): 9:00 11:00 12:00

No books, notes, calculators, or any electronic devices. If you need more space to answer a problem then use the back of one of these pages. **Do not separate the pages!** Indicate where the answer to each part of each problem is located. Cross out work that you do not want considered. **Your reasoning must be given for full credit!** Good luck!

Please copy and sign the University Honor Pledge below.
Please print your name on each page.

University Honor Pledge: *I pledge on my honor that I have not given or received any unauthorized assistance on this examination.* ____

____

____

Signature:____

Problem 1:	________/10	Problem 2:	________/10
Problem 3:	________/5	Problem 4:	________/5
Problem 5:	________/10	Problem 6:	________/10
Problem 7:	________/10	Problem 8:	________/10
Problem 9:	________/5	Problem 10:	________/7
Problem 11:	________/10	Problem 12:	________/8
		Total Score:	________/100
		Grade:	________

Name:____

(1) [10] Find an explicit solution for the following initial-value problems and give its interval of definition.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 3t^2(2x - x^2)\,, \qquad x(0) = 1\,.$$

(2) [10] Find an explicit solution for the following initial-value problems and give its interval of definition.

$$(z^2 - 4)\frac{\mathrm{d}y}{\mathrm{d}z} + 6zy = \frac{3}{z^2 - 4}\,, \qquad y(0) = 3\,.$$

Name:____

(3) [5] Sketch the graph that would be produced by the following Matlab commands.

```
[X, Y] = meshgrid(−4:0.1:4,−4:0.1:4)
contour(X, Y, X.*Y, [−8, −4, −2])
axis square
```

(4) [5] Give the interval of definition for the solution of the initial-value problem

$$\frac{\mathrm{d}w}{\mathrm{d}t} + \frac{\cos(t)}{t^2 - 25}\, w = \frac{e^t}{\sin(t)}\,, \qquad w(-4) = 5\,.$$

(Do not solve the equation to answer this question, but give reasoning!)

Name:____

(5) [10] Consider the differential equation

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{(u^2-4)(u+6)^2}{(u^2+4)(u-6)} \,.$$

(a) [7] Sketch its phase-line portrait over the interval $-8 \leq u \leq 8$. Identify points where solutions are undefined with a $\circ$. Identify stationary points with a $\bullet$ and classify each as being either stable, unstable, or semistable.

(b) [3] For each stationary point identify the set of initial-values $u(0)$ such that the solution $u(t)$ converges to that stationary point as $t \to -\infty$.

(6) [10] Determine if the following differential form is exact. If it is then find an implicit general solution. Otherwise find an integrating factor. (You do not need to find a general solution in the last case.)

$$\big(\cos(x) - \sin(x+y)\big)\,\mathrm{d}x + \big(e^y - \sin(x+y)\big)\,\mathrm{d}y = 0\,.$$

Name:____________________________________

(7) [10] Consider the following MATLAB function m-file.

```
function [t,x] = solveit(tI, xI, tF, n)

t = zeros(n + 1, 1); x = zeros(n + 1, 1);
t(1) = tI; x(1) = xI; h = (tF - tI)/n; hhalf = h/2;
for k = 1:n
thalf = t(k) + hhalf; t(k + 1) = t(k) + h;
fnow = (t(k))^2 + exp(t(k)*x(k)); xhalf = x(k) + hhalf*fnow;
fhalf = (thalf)^2 + exp(thalf*xhalf); x(k + 1) = x(k) + h*fhalf;
end
```

Suppose the input values are tI = 1, xI = 0, tF = 11, and n = 50.

(a) [4] What initial-value problem is being approximated numerically?
(b) [1] What is the numerical method being used?
(c) [1] What is the step size?
(d) [4] What will be the output values of t(2) and x(2)?

(8) [10] Determine if the following differential form is exact. If it is then find an implicit general solution. Otherwise find an integrating factor. (You do not need to find a general solution in the last case.)

$$x^2\,\mathrm{d}x + (x^3 + y^6 + 2y^5)\,\mathrm{d}y = 0\,.$$

Name:______________________________________

(9) [5] A tank has a square base with 3 meter edges, a height of 5 meters, and an open top. It is initially empty when water begins to fill it at a rate of 9 liters per minute. The water also drains from the tank through a hole in its bottom at a rate of $4\sqrt{h}$ liters per minute where $h(t)$ is the height of the water in the tank in meters.

(a) [4] Give an initial-value problem that governs $h(t)$. (Recall 1 $\text{m}^3 = 1000$ lit.) (Do not solve the initial-value problem!)

(b) [1] Does the tank overflow? (Why or why not?)

(10) [7] In the absence of predators the population of mosquitoes in a certain area would increase at a rate proportional to its current population such that it would triple every seven weeks. There are 320,000 mosquitoes in the area when a flock of birds arrives that eats 50,000 mosquitoes per week.

(a) [5] Give an initial-value problem that governs $M(t)$, the number of mosquitoes in the area after the flock of birds arrives. (Do not solve the initial-value problem!)

(b) [2] Is the flock large enough to control the mosquitoes? (Why or why not?)

Name:______________________________________

(11) [10] A puck with initial velocity $v_o > 0$ begins to slide on a surface that imparts a position-dependent frictional drag. Its position $x(t)$ is governed by the initial-value problem

$$\ddot{x} = -e^{-x}\dot{x}\,, \qquad x(0) = 0, \quad \dot{x}(0) = v_o > 0\,.$$

(a) [8] Solve the auxiliary equation and write down the resulting reduced equation.
(b) [2] Find the smallest initial velocity v_o for which $x(t) \to \infty$ as $t \to \infty$.

(12) [8] Suppose you have used a numerical method to approximate the solution of an initial-value problem over the time interval $[2, 10]$ with 800 uniform time steps. What step size is needed to reduce the global error of your approximation by a factor of $\frac{1}{256}$ if the method you had used was each of the following? (Notice that $256 = 4^4$.)

(a) explicit Euler method

(b) Runge-Kutta method

(c) Runge-midpoint method

(d) Runge-trapezoidal method