Quiz 9, Math 246, Professor David Levermore
Tuesday, 13 November 2018

Your Name:

Discussion Instructor (circle one): Sid Sharma Anqi Ye
Discussion Time (circle one): 8:00 9:00 10:00

No books, notes, calculators, or any electronic devices.
Show your reasoning for full credit. Good luck!

(1) [5] Consider the vector-valued functions $\mathbf{x}_1(t) = \begin{pmatrix} 1 \\ e^t \end{pmatrix}$, $\mathbf{x}_2(t) = \begin{pmatrix} t^2 \\ e^t \end{pmatrix}$.

(a) [2] Compute their Wronskian $\mathrm{Wr}[\mathbf{x}_1, \mathbf{x}_2](t)$.

(b) [3] Find $\mathbf{A}(t)$ such that $\mathbf{x}_1$, $\mathbf{x}_2$ is a fundamental set of solutions to $\mathbf{x}' = \mathbf{A}(t)\mathbf{x}$.

(2) [4] Let $\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 5 & 3 \end{pmatrix}$. Compute $e^{t\mathbf{A}}$.

(3) [1] Suppose that $e^{t\mathbf{A}} = e^{2t} \begin{pmatrix} \cosh(3t) & \frac{1}{3}\sinh(3t) \\ 3\sinh(3t) & \cosh(3t) \end{pmatrix}$.

Compute the Green matrix $\mathbf{G}(t, s)$ associated with $\mathbf{x}' = \mathbf{A}\mathbf{x}$.