Math 246, Professor David Levermore
Group Work Exercises for Discussion
Wednesday, 24 October 2018

Group Work Exercises related to Exam 2 [5]

These exercises are based upon Problem 4 of Exam 2. It considered a linear ordinary differential operator L with constant coefficients such that all of the roots of its characteristic polynomial (listed with their multiplicities) are $-2+i3$, $-2+i3$, $-2-i3$, $-2-i3$, 3, 3, 0, 0, 0, 0. Assume that L is in normal form.

- Give its characteristic polynomial $p(z)$. (Leave it in factored form!)
- Give L. (Leave it in factored form!)
- Give the degree d, characteristic $\mu + i\nu$, and multiplicity m for the forcing of the nonhomogeneous equation $\mathrm{L}w = t^2 e^{3t}$.
- Consider the nonhomogeneous equation $\mathrm{L}w = t^2 e^{3t}$. Give the form of the particular solution used by the method of Undetermined Coefficients. (Do not do more!)
- Consider the nonhomogeneous equation $\mathrm{L}w = t^2 e^{3t}$. Write down the derivatives of the Key Identity that need to be evaluated at the characteristic by the method of Key Identity Evaluations. (Do not do more!)

Group Work Exercises related to Quiz 7 [5]

- Use the definition of the Laplace transform to compute $\mathcal{L}[f](s)$ for the function $f(t) = u(t-4)e^{-3t}$, where u is the unit step function.
- What is the exponential order of $f(t) = u(t-4)e^{-3t}$ as $t \to \infty$?
- How might we have guessed that $\mathcal{L}[f](s)$ is defined only for $s > -3$?
- If $h(t) = u(t-5)t\,e^{-4t}\sin(3t)$ then for what values of s will $\mathcal{L}[h](s)$ be defined?
- Use the short table of Laplace transforms below to compute $\mathcal{L}[h](s)$.

A Short Table of Laplace Transforms

$$\mathcal{L}[t^n e^{at}](s) = \frac{n!}{(s-a)^{n+1}} \qquad \text{for } s > a\,.$$

$$\mathcal{L}[e^{at}\cos(bt)](s) = \frac{s-a}{(s-a)^2+b^2} \qquad \text{for } s > a\,.$$

$$\mathcal{L}[e^{at}\sin(bt)](s) = \frac{b}{(s-a)^2+b^2} \qquad \text{for } s > a\,.$$

$$\mathcal{L}[t^n j(t)](s) = (-1)^n J^{(n)}(s) \qquad \text{where } J(s) = \mathcal{L}[j(t)](s)\,.$$

$$\mathcal{L}[e^{at} j(t)](s) = J(s-a) \qquad \text{where } J(s) = \mathcal{L}[j(t)](s)\,.$$

$$\mathcal{L}[u(t-c)j(t-c)](s) = e^{-cs}J(s) \qquad \text{where } J(s) = \mathcal{L}[j(t)](s) \text{ and } u \text{ is the unit step function}\,.$$