

Math 246, Professor David Levermore
Group Work Exercises for Discussion
Wednesday, 26 September 2018

First Set of Group Work Exercises [5]

These questions are based upon Problem 2a of Exam 1, which was to find the explicit solution and give its interval of definition for the initial-value problem

$$\frac{dx}{dt} = 3t^2(x - x^2), \quad x(0) = 2.$$

Based upon the posted solution of this problem, address the following questions.

- (1) How does the solution $x(t)$ behave as $t \rightarrow \infty$?
- (2) How does the solution $x(t)$ behave as $t \rightarrow -(\log(2))^{\frac{1}{3}}$?
- (3) How do $x(t)$ and its interval of definition change if the initial condition is $x(3) = 2$? Describe how $x(t)$ behaves as t approaches each endpoint of its interval of definition.
- (4) How do $x(t)$ and its interval of definition change if the initial condition is $x(0) = -1$? Describe how $x(t)$ behaves as t approaches each endpoint of its interval of definition.
- (5) How do $x(t)$ and its interval of definition change if the initial condition is $x(0) = \frac{1}{2}$? Describe how $x(t)$ behaves as t approaches each endpoint of its interval of definition.

Second Set of Group Work Exercises [5]

- (1) Verify that $U_1(t) = e^{-2t}$ and $U_2(t) = t e^{-2t}$ solve

$$u'' + 4u' + 4u = 0.$$

- (2) Solve the initial-value problems

$$u'' + 4u' + 4u = 0, \quad u(0) = 1, \quad u'(0) = 0;$$

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- (3) Solve the general initial-value problem

$$u'' + 4u' + 4u = 0, \quad u(0) = u_0, \quad u'(0) = u_1.$$

- (4) Compute the $\text{Wr}[U_1, U_2](t)$. (Evaluate the determinant and simplify.)
- (5) Verify that $w(t) = \text{Wr}[U_1, U_2](t)$ solves the homogeneous linear first-order equation given by the Abel Theorem.