

**Thirteenth Homework: MATH 410**  
**Due Thursday, 30 November 2017**

1. Exercise 1 of Section 6.5 in the text.
2. Exercise 5 of Section 6.5 in the text.
3. Exercise 1 of Section 6.6 in the text.
4. Exercise 3 of Section 6.6 in the text.
5. Exercise 7 of Section 6.6 in the text.
6. Exercise 3 of Section 7.2 in the text.
7. Exercise 4 of Section 7.2 in the text.
8. Exercise 5 of Section 7.2 in the text.
9. Exercise 9 of Section 7.2 in the text.
10. Let  $f : [a, b] \rightarrow \mathbb{R}$ . Let  $F : [a, b] \rightarrow \mathbb{R}$  be a primitive of  $f$  over  $[a, b]$ . Let  $g : [a, b] \rightarrow \mathbb{R}$  such that  $g(x) = f(x)$  at all but a finite number of points of  $[a, b]$ . Show that  $F$  is also a primitive of  $g$  over  $[a, b]$ .
11. Let  $f : [0, 3] \rightarrow \mathbb{R}$  be defined by

$$f(x) = \begin{cases} x & \text{for } 0 \leq x < 1, \\ -x & \text{for } 1 \leq x < 2, \\ 1 & \text{for } 2 \leq x \leq 3. \end{cases}$$

- Find  $F$ , the primitive of  $f$  over  $[0, 3]$  specified by  $F(0) = 1$ .
12. The assumption that  $G$  is increasing over  $[a, b]$  in Proposition 11.2 of the Notes can be weakened to the assumption that  $G$  is nondecreasing over  $[a, b]$ . Prove this. The proof can be very similar to that given for Proposition 11.2 except you will have to work harder to show that  $F(G)$  is a primitive of  $f(G)g$  over  $[a, b]$ . Specifically, because  $G^{-1}$  may not exist, you will need to replace the partition  $G^{-1}(P)$  in the proof of Proposition 11.2 with a more complicated partition.