

**Quiz 1 Solutions, Math 246, Professor David Levermore**  
**Thursday, 31 August 2017**

(1) [4] For each of the following ordinary differential equations, determine its order and whether it is linear or nonlinear. If it is nonlinear, circle a term that makes it so.

(a)  $v'''' + \sin(v)v' = 5e^t$                       **Solution.** fourth order, nonlinear,  $\sin(v)v'$ .

(b)  $k''' + \cos(t)k'' = 3k + \sin(3t)$                       **Solution.** third order, linear.

(2) [4] Solve the initial-value problem

$$z \frac{du}{dz} + 4u = 12z^2, \quad u(-1) = 5.$$

**Solution.** This equation is linear. Its normal form is

$$\frac{du}{dz} + \frac{4}{z}u = 12z.$$

An integrating factor is  $e^{A(z)}$  where  $A'(z) = 4/z$ . Setting  $A(z) = 4 \log(|z|)$ , we obtain  $e^{A(z)} = e^{4 \log(|z|)} = z^4$ . Hence, the problem has the integrating factor form

$$\frac{d}{dz}(z^4u) = z^4 \cdot (12z) = 12z^5.$$

Integrating both sides yields

$$z^4u = 2z^6 + c.$$

Imposing the initial condition gives

$$(-1)^4 \cdot 5 = 2 \cdot (-1)^6 + c,$$

whereby  $c = 3$ . Therefore the solution is

$$u = \frac{2z^6 + 3}{z^4}.$$

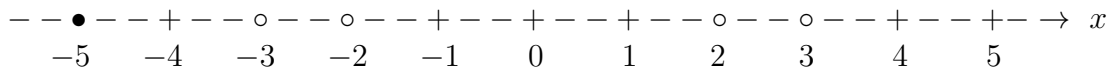
**Remark.** The interval of definition for this solution is  $(-\infty, 0)$ . Do you see why?

(3) [2] What is the interval of definition for the solution of the initial-value problem

$$\frac{dw}{dx} + \frac{\sin(x)}{x^2 - 9}w = \frac{e^x}{x^2 - 4}, \quad w(-5) = 7.$$

(You do not have to solve the differential equation to answer this question!)

**Solution.** This is a nonhomogeneous linear equation that is already in normal form. The coefficient  $\sin(x)/(x^2 - 9)$  is undefined at  $x = \pm 3$  and is continuous elsewhere. The forcing  $e^x/(x^2 - 4)$  is undefined at  $x = \pm 2$  and is continuous elsewhere. The initial time is  $x = -5$ . This can be pictured on the  $x$ -axis as follows.



Therefore the interval of definition for the solution is  $(-\infty, -3)$  because:

- the initial time  $x = -5$  is in  $(-\infty, -3)$ ,
- the coefficient is undefined at  $x = -3$ ,
- the coefficient and forcing are both continuous over  $(-\infty, -3)$ .

**Question.** How does the answer change if the initial condition is  $w(1) = -4$ ?