First Homework: AMSC/CMSC 460
Due Tuesday, 6 September 2016

1. Show that there exists a real solution to $x^4-2x^3-4x^2+4x+2=0$ that lies in $[0,2]$. Use the bisection method starting with the interval $[0,2]$ to approximate such a solution to within 10^{-2}. Print out each interval $[a_n,b_n]$ that you compute.
2. Show that there exists a unique real solution to $4x = e^x$ that lies in $[0,1]$. Use the bisection method starting with the interval $[0,1]$ to approximate this solution to within 10^{-3}. Print out each interval $[a_n,b_n]$ that you compute.
3. Use the Newton method to approximate the solution of $4x = e^x$ that lies in $[0,1]$ to within 10^{-5}. Which is the better choice for an initial guess: $x_0 = 0$ or $x_0 = 1$? Why? Print out your initial guess x_0 and each Newton iterate that you compute. How does the number of Newton iterates compare to the number of steps of the bisection method that you used in problem 2?
4. Use the bisection method to find the smallest positive solution of $x = \tan(x)$ to within 10^{-3}. Justify your choice of initial interval $[a_0,b_0]$. Print out each interval $[a_n,b_n]$ that you compute.
5. Use the Newton method to find the smallest positive solution of $x = \tan(x)$ to within 10^{-5}. Use as your initial guess either $x_0 = a_0$ or $x_0 = b_0$ where $[a_0,b_0]$ was your initial interval in problem 4. Print out your initial guess x_0 and each Newton iterate that you compute. How does the number of Newton iterates compare to the number of steps of the bisection method that you used in problem 4?
6. The following fix-point methods are proposed to compute $21^{\frac{1}{3}}$ with an initial guess of $x_0 = 3$. Rank them in order based on their apparent speed of convergence.

a. $$x_{n+1} = \frac{20x_n + 21/x_n^2}{21},$$
b. $$x_{n+1} = x_n - \frac{x_n^3 - 21}{3x_n^2},$$
c. $$x_{n+1} = x_n - \frac{x_n^4 - 21x_n}{x_n^2 - 21},$$
d. $$x_{n+1} = \frac{x_n^2 + 21/x_n}{2x_n}.$$

Can you explain your results?