Quiz 11 Solutions, Math 246, Professor David Levermore
Thursday, 5 December 2013

(1) [5] A 2×2 matrix $\mathbf{A}$ has the eigenpairs

$$\left(-2\,,\begin{pmatrix}1\\-1\end{pmatrix}\right)\,,\qquad \left(-1\,,\begin{pmatrix}2\\1\end{pmatrix}\right)\,.$$

Sketch a phase-plane portrait that indicates typical orbits for the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$. Identify its type. Classify the origin as either attracting, stable but not attracting, unstable but not repelling, or repelling.

Solution. Because $\mathbf{A}$ has two negative eigenvalues, the phase portrait is a *nodal sink*. The origin is thereby *attracting*. The phase portrait should show one orbit that approaches the origin along each half of the lines $y = -x$ and $y = \frac{1}{2}x$. The phase portrait should indicate that every other orbit approaches the origin tangent to the line $y = \frac{1}{2}x$.

(2) [5] Sketch a phase-plane portrait that indicates typical orbits for the system

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3 & 2\\-5 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}\,.$$

Identify its type. Classify the origin as either attracting, stable but not attracting, unstable but not repelling, or repelling.

Solution. The characteristic polynomial of the given matrix $\mathbf{A}$ is

$$p(z) = z^2 - \mathrm{tr}(\mathbf{A})z + \det(\mathbf{A}) = z^2 - 4z + 13 = (z-2)^2 + 9\,.$$

We see that $\mu = 2$ and $\delta = -9$. There are no real eigenpairs. Because $\mu = 2 > 0$, $\delta = -9 < 0$, and $a_{21} = -5 < 0$ the phase portrait is a *clockwise spiral source*. The origin is thereby *repelling*. The phase portrait should indicate a family of clockwise spiral orbits that emerge from the origin.