

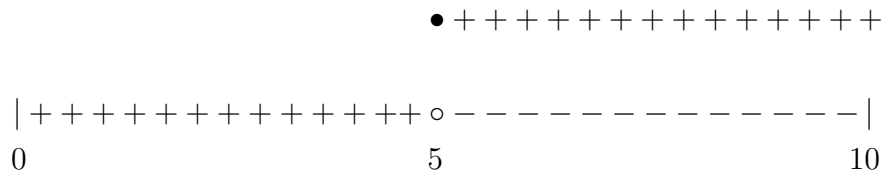
Quiz 8, Math 246, Professor David Levermore
Tuesday, 6 November 2012

- (1) [2] Sketch a graph of the function $u(t - 5)$ over $0 \leq t \leq 10$.

Solution. Because

$$u(t - 5) = \begin{cases} 0 & \text{for } 0 \leq t < 5, \\ 1 & \text{for } 5 \leq t \leq \infty, \end{cases}$$

the graph of $u(t - 5)$ over $0 \leq t \leq 10$ is indicated below by the plus signs.



- (2) [3] Give the exponential order α as $t \rightarrow \infty$ of the following functions.

(a) e^{7t}

Solution. $\alpha = 7$

(b) $t^2 \sin(3t)$

Solution. $\alpha = 0$

(c) $t^3 e^{-2t}$

Solution. $\alpha = -2$

- (3) [5] Use the definition of the Laplace transform to compute $\mathcal{L}[f](s)$ for the function $f(t) = e^{-3t}u(t - 5)$, where u is the unit step function.

Solution. By the definitions of the Laplace transform and the unit step function

$$\begin{aligned} \mathcal{L}[f](s) &= \lim_{T \rightarrow \infty} \int_0^T e^{-st} f(t) dt = \lim_{T \rightarrow \infty} \int_0^T e^{-st} e^{-3t} u(t - 5) dt \\ &= \lim_{T \rightarrow \infty} \int_5^T e^{-st} e^{-3t} dt = \lim_{T \rightarrow \infty} \int_5^T e^{-(s+3)t} dt. \end{aligned}$$

For $s \leq -3$ the above limit diverges because $e^{-(s+3)t} \geq 1$. For $s > -3$

$$\int_5^T e^{-(s+3)t} dt = -\frac{e^{-(s+3)t}}{s+3} \Big|_5^T = \frac{e^{-(s+3)5}}{s+3} - \frac{e^{-(s+3)T}}{s+3},$$

whereby

$$\mathcal{L}[f](s) = \lim_{T \rightarrow \infty} \left[\frac{e^{-(s+3)5}}{s+3} - \frac{e^{-(s+3)T}}{s+3} \right] = \frac{e^{-(s+3)5}}{s+3} \quad \text{for } s > -3.$$