Quiz 3 Solutions, Math 246, Professor David Levermore
Tuesday, 18 September 2012

(1) [4] In the absence of predators the population of mosquitoes in a certain area would increase at a rate proportional to its current population such that it would double every five weeks. There are 180,000 mosquitoes in the area when a flock of birds arrives that eats 45,000 mosquitoes per week. Write down an initial-value problem that governs the population of mosquitoes in the area after the flock of birds arrives. (Do not solve the initial-value problem!)

Solution. Let $M(t)$ be the number of mosquitoes at time t weeks. Doubling every five weeks gives a growth factor of $2^{\frac{t}{5}} = (e^{\log(2)})^{\frac{t}{5}} = e^{\frac{1}{5}\log(2)t}$, which implies a growth rate of $\frac{1}{5}\log(2)$. The initial-value problem that M satisfies is therefore

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \tfrac{1}{5}\log(2)M - 45,000\,, \qquad M(0) = 180,000\,.$$

(2) [4] A student borrows \$7000 at an interest rate of 9% per year that is compounded continuously. Assume that the student makes payments continuously at a constant rate of \$1500 per year. Let $B(t)$ denote the balance of the loan at t years. Write down an initial-value problem that governs $B(t)$ for so long as $B(t) \geq 0$.

Solution. Because an interest rate of 9% per year is an interest rate of .09 per year, the initial-value problem that governs $B(t)$ is

$$\frac{\mathrm{d}B}{\mathrm{d}t} = .09B - 1500\,, \qquad B(0) = 7000\,.$$

(3) [2] Consider the initial-value problem

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 4x - e^x \qquad x(1) = 3\,.$$

If you approximate $x(5)$ by using 100 steps of the explicit Euler method with uniform time steps, what is the size of the time step h?

Solution. Because the initial time $t_I = 1$, the final time $t_F = 5$, and the number of time steps $N = 100$, the size of the time step is

$$h = \frac{t_F - t_I}{N} = \frac{5-1}{100} = \frac{4}{100} = .04\,.$$