

**Quiz 2 Solutions, Math 246, Professor David Levermore**  
**Tuesday, 11 September 2012**

- (1) [5] Find the solution of the initial-value problem

$$\frac{dx}{dt} = t^2 e^{-3x}, \quad x(0) = x_I \quad \text{for some } x_I \text{ in } (-\infty, \infty).$$

Give its interval of definition in terms of  $x_I$ .

**Solution.** This equation is separable. Its separated differential form is  $e^{3x} dx = t^3 dt$ , whereby

$$\int e^{3x} dx = \int t^3 dt.$$

By integrating both sides gives  $\frac{1}{3}e^{3x} = \frac{1}{3}t^3 + c$ . The initial condition then implies that  $\frac{1}{3}e^{3x_I} = \frac{1}{3}0^3 + c$ , whereby  $c = \frac{1}{3}e^{3x_I}$ . The solution is thereby given implicitly by

$$e^{3x} = t^3 + e^{3x_I}.$$

Whenever  $t^3 + e^{3x_I} > 0$  this can be solved explicitly for  $x$  to obtain

$$x = \frac{1}{3} \log(t^3 + e^{3x_I}).$$

The condition  $t^3 + e^{3x_I} > 0$  implies that  $t^3 > -e^{3x_I}$ , which implies that  $t > -e^{x_I}$ . Therefore the interval of definition is  $(-e^{x_I}, \infty)$ .

- (2) [5] Sketch the phase-line portrait for the equation

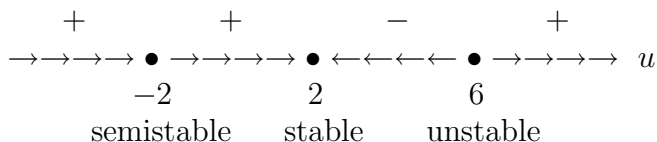
$$\frac{du}{dt} = \frac{(u+2)^2(u-2)(u-6)}{4+u^2}.$$

Identify every stationary (equilibrium) point as either stable, unstable, or semistable. (You do not have to find the solution!)

**Solution.** The stationary points are found by setting

$$(u+2)^2(u-2)(u-6) = 0.$$

Therefore the stationary points are  $u = -2$ ,  $u = 2$ , and  $u = 6$ . The phase-line portrait is



**Remark.** If  $u(0)$  lies within the interval  $[-2, 6]$  then the interval of definition of the solution  $u(t)$  is  $(-\infty, \infty)$ . Do you see why?