

Homework Problems on Natural Fundamental Sets
Fall 2012, Math 246, Professor David Levermore

1. Given that e^{10t} and e^{-2t} is a fundamental set of solutions to $x'' - 8x' - 20x = 0$, find the solution $X(t)$ to the general initial-value problem

$$x'' - 8x' - 20x = 0, \quad x(0) = x_0, \quad x'(0) = x_1.$$

Find the natural fundamental set of solutions associated with the initial time 0.

2. Given that $e^{2t} \cos(t)$ and $e^{2t} \sin(t)$ is a fundamental set of solutions to $y'' - 4y' + 5y = 0$, find the solution $Y(t)$ to the general initial-value problem

$$y'' - 4y' + 5y = 0, \quad y(0) = y_0, \quad y'(0) = y_1.$$

Find the natural fundamental set of solutions associated with the initial time 0.

3. Given that t and t^3 is a fundamental set of solutions to $t^2 z'' - 3tz' + 3z = 0$ for $t > 0$, find the solution $Z(t)$ to the general initial-value problem

$$t^2 z'' - 3tz' + 3z = 0, \quad z(2) = z_0, \quad z'(2) = z_1.$$

Find the natural fundamental set of solutions associated with the initial time 2.