

Homework Problems on the Green Function Method II
Fall 2012, Math 246, Professor David Levermore

1. Compute the Green function $G(t, s)$ for the differential operator $L(t)$ defined by

$$L(t)y = D^2y - 2tDy + (t^2 - 1)y,$$

given that $e^{\frac{1}{2}t^2}$ and $te^{\frac{1}{2}t^2}$ solve the homogeneous equation $L(t)y = 0$. Use the result to solve the initial-value problem

$$y'' - 2ty' + (t^2 - 1)y = t^2e^{\frac{1}{2}t^2}, \quad y(0) = y'(0) = 0.$$

2. Compute the Green function $G(t, s)$ for the differential operator $L(t)$ defined by

$$L(t)y = tD^2y + (t - 1)Dy - y,$$

given that $t - 1$ and e^{-t} solve the homogeneous equation $L(t)y = 0$. Use the result to solve the initial-value problem

$$ty'' + (t - 1)y' - y = 2t^3, \quad y(1) = y'(1) = 0.$$

Remark: You can also solve these problems using variation of parameters.