

Solutions to Homework Problems on the Green Function Method I
Fall 2012, Math 246, Professor David Levermore

1. Compute the Green functions associated with the following differential operators L .

a) $L = D^2 + 8D - 20$

b) $L = D^2 + 8D + 20$

c) $L = D^4 + 13D^2 + 36$

Solutions. The final answers are:

a) $g(t) = \frac{1}{12}(e^{2t} - e^{-10t}),$

b) $g(t) = \frac{1}{2}e^{-4t} \sin(2t),$

c) $g(t) = \frac{1}{15} \sin(3t) - \frac{1}{10} \sin(2t).$

2. Use the Green function method to find a general solution of the following equations.

a) $D^2y - Dy - 2y = \frac{1}{e^t + 1}$

b) $D^2y + y = \tan(t)$

c) $D^2y + y = \frac{1}{16 + 9 \cos(t)^2}$

Solutions. The answers expressed first as definite integrals and then explicitly are:

a)
$$y = c_1 e^{2t} + c_2 e^{-t} + \frac{1}{3} e^{2t} \int_0^t \frac{e^{-2s}}{e^s + 1} ds - \frac{1}{3} e^{-t} \int_0^t \frac{e^s}{e^s + 1} ds$$

$$= c_1 e^{2t} + c_2 e^{-t} + \frac{1}{3} e^{2t} \left[-\frac{1}{2}(1 - e^{-t})^2 - \log\left(\frac{e^{-t} + 1}{2}\right) \right] - \frac{1}{3} e^{-t} \log\left(\frac{e^t + 1}{2}\right),$$

b)
$$y = c_1 \cos(t) + c_2 \sin(t) + \sin(t) \int_0^t \cos(s) \tan(s) ds - \cos(t) \int_0^t \sin(s) \tan(s) ds$$

$$= c_1 \cos(t) + c_2 \sin(t) + \sin(t) - \cos(t) \frac{1}{2} \log\left(\frac{1 + \sin(t)}{1 - \sin(t)}\right),$$

c)
$$y = c_1 \cos(t) + c_2 \sin(t) + \sin(t) \int_0^t \frac{\cos(s)}{16 + 9 \cos(s)^2} ds - \cos(t) \int_0^t \frac{\sin(s)}{16 + 9 \cos(s)^2} ds$$

$$= c_1 \cos(t) + c_2 \sin(t) + \frac{\sin(t)}{30} \log\left(\frac{5 + 3 \sin(t)}{5 - 3 \sin(t)}\right)$$

$$+ \frac{\cos(t)}{12} \left[\tan^{-1}\left(\frac{3}{4} \cos(t)\right) - \tan^{-1}\left(\frac{3}{4}\right) \right].$$