Quiz 1 Solutions, Math 220, Professor David Levermore
Friday, 3 September 2010

(1) [3] Factor $x^2 - 3x - 18$.

Solution. The roots of $x^2-3x-18$ are 6 and -3, so that $x^2-3x-18 = (x-6)(x+3)$. These roots can found either by trying the factors of -18, by the quadratic formula, or by completing the square. The factors of -18 are ±1, ±2, ±3, ±6, and ±9; one easily sees that ±1 and ±2 are not roots while -3 is a root because

$$(-3)^2 - 3(-3) - 18 = 9 + 9 - 18 = 0\,.$$

The other root is found by using either the fact that the sum of the roots is 3 or the fact that the product of the roots is -18. The quadratic formula yields the roots

$$\frac{3 \pm \sqrt{3^2 - 4(-18)}}{2} = \frac{3 \pm \sqrt{9+72}}{2} = \frac{3 \pm \sqrt{81}}{2} = \frac{3 \pm 9}{2}\,.$$

Completing the square produces the factors

$$\begin{aligned} x^2 - 3x - 18 &= (x - \tfrac{3}{2})^2 - 18 - (\tfrac{3}{2})^2 = (x - \tfrac{3}{2})^2 - \tfrac{72}{4} - \tfrac{9}{4} \\ &= (x - \tfrac{3}{2})^2 - \tfrac{81}{4} = (x - \tfrac{3}{2} - \tfrac{9}{2})(x - \tfrac{3}{2} + \tfrac{9}{2}) = (x-6)(x+3)\,. \end{aligned}$$

(2) [6] Let $f(x) = x^2 - 2x + 5$ and $g(x) = x - 3$.

(a) Evaluate $f(x) + g(x)$.

Solution.

$$f(x) + g(x) = (x^2 - 2x + 5) + (x - 3) = x^2 - x + 2\,.$$

(b) Evaluate $f(g(x))$.

Solution.

$$\begin{aligned} f(g(x)) &= g(x)^2 - 2g(x) + 5 = (x-3)^2 - 2(x-3) + 5 \\ &= x^2 - 6x + 9 - 2x + 6 + 5 = x^2 - 8x + 20\,. \end{aligned}$$

(3) [1] Is the point $(2, -1)$ on the graph of the function $h(x) = 3x$?

Solution. No, because $h(2) = 3 \cdot 2 = 6 \neq -1$.