Thirteenth Homework: MATH 410
Due Tuesday, 1 December 2009

1. Exercise 1 of Section 6.5 in the text.
2. Exercise 5 of Section 6.5 in the text.
3. Exercise 1 of Section 6.6 in the text.
4. Exercise 3 of Section 6.6 in the text.
5. Exercise 7 of Section 6.6 in the text.
6. Exercise 3 of Section 7.2 in the text.
7. Exercise 4 of Section 7.2 in the text.
8. Exercise 5 of Section 7.2 in the text.
9. Exercise 9 of Section 7.2 in the text.
10. Let $f : [a, b] \to \mathbb{R}$. Let $F : [a, b] \to \mathbb{R}$ be a primitive of f over $[a, b]$. Let $g : [a, b] \to \mathbb{R}$ such that $g(x) = f(x)$ at all but a finite number of points of $[a, b]$. Show that F is also a primitive of g over $[a, b]$.
11. Let $f : [0, 3] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x & \text{for } 0 \le x < 1, \\ -x & \text{for } 1 \le x < 2, \\ 1 & \text{for } 2 \le x \le 3. \end{cases}$$

Find F, the primitive of f over $[0, 3]$ specified by $F(0) = 1$.
12. The assumption that G is increasing over $[a, b]$ in Proposition 3.2 of the Notes can be weakened to the assumption that G is nondecreasing over $[a, b]$. Prove this. The proof can be very similar to that given for Proposition 3.2 except you will have to work harder to show that $F(G)$ is a primitive of $f(G)g$ over $[a, b]$. Specifically, because G^{-1} may not exist, you will need to replace the partition $G^{-1}(P)$ in the proof of Proposition 3.2 with a more complicated partition.