

Fourth and Fifth Homework: MATH 410
Due in class, Monday, 9 October 2006

1. Exercises 1e,f,g of Section 9.1 in the text.
2. Exercise 2 of Section 9.1 in the text.
3. Exercise 3 of Section 9.1 in the text.
4. Exercise 4 of Section 9.1 in the text.
5. Determine all the values of $a \in \mathbb{R}$ for which the following formal infinite series converge. Give your reasoning.

(a)
$$\sum_{n=2}^{\infty} \frac{1}{\log(n)} a^n$$

(b)
$$\sum_{k=0}^{\infty} \left(\frac{2k+3}{k^4+1} \right)^a$$

(c)
$$\sum_{m=1}^{\infty} \frac{1}{m^2} (2 + (-1)^m)^m a^m$$

6. Let $\{a_k\}_{k \in \mathbb{Z}_+}$ be a positive, nonincreasing sequence. Prove

$$\sum_{m=1}^{\infty} 5^m a_{5^m} \text{ converges} \quad \implies \quad \sum_{k=1}^{\infty} a_k \text{ converges.}$$