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Expected coalescing length of displacement loading collinear microcracks

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Abstract

Under displacement loading, brittle material softens due to the coalescence of statistically distributed microcracks. Microcracks propagate, coalesce, and eventually reach a stable configuration. The situation is simulated in the present work by a collinear microcrack configuration with the attention focused on the expected crack length. The results indicate that the expected crack length increases by higher microcrack density, less microcrack randomness, and a larger specimen size (scale effect).

1. Introduction

Stress or displacement loading leads to different responses for brittle materials containing distributed microcracks. Coalescence of microcracks under stress loading is dictated by the critical linkage. Under displacement loading, however, softening by microcrack coalescence plays an important role. Hence, the expected length of coalescing microcracks is a concern, and will be estimated statistically.

2. Configuration

Consider the simple case of collinear microcracks under uniaxial displacement Δ normal to the cracks as shown in Fig. 1(a). All microcracks have the same length of $2a_0$, but are separated by ligaments whose size c is governed by a normal distribution $p_0(c)$.

Assuming a self-similar coalescing process of microcracks. Fig. 1(b) shows a representative configuration that deals with the linkage of two neighboring microcracks. Displacement loading is invoked by a pair of continuously distributed dislocations. The linking process is characterized by the interaction of neighboring microcracks and two continuously distributed dislocations. In the subsequent calculations, the representative configuration will have a height of $H = 8a_0$ and leave two ligaments of $4a_0$ at both sides.

3. Energy ratio

For a brittle material, the linking process may be treated by the pop-through of the ligaments, without considering the acceleration or deceleration of crack tips during the connection. Defined is an energy ratio to characterize the linking process

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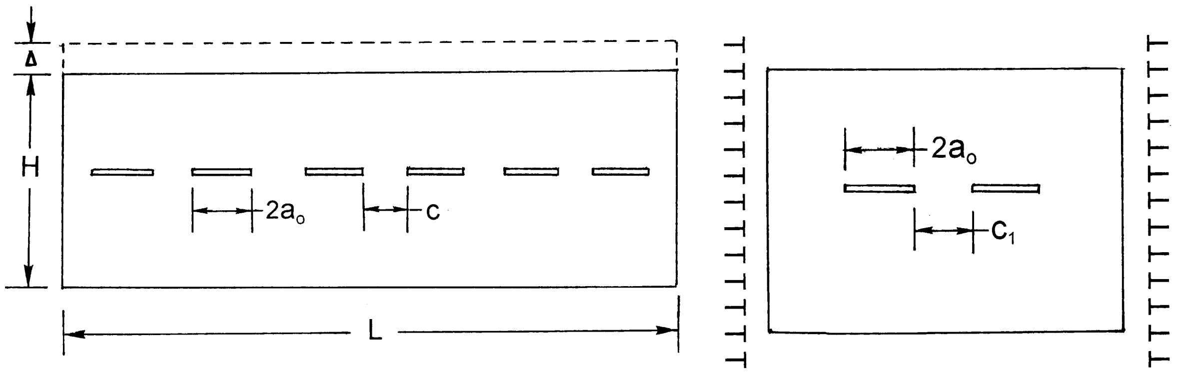

Fig. 1. (a) (left) Collinear microcracks configuration; (b) (right) representative configuration.

$$R = \frac{\Delta \Pi}{2c\gamma_s}, \tag{1}$$

where $\Delta \Pi$ indicates the release of potential energy in linking the neighboring microcracks and γ_s denotes the surface tension of the brittle matrix. The ratio R describes the competition between the potential energy release and the newly created crack surface energy in each coalescing step, and serves as the excessive driving force for microcrack propagation. The ratio R increases with the coalescence probability.

The potential energy contributed from the ith microcrack is given by

$$\Pi^{(i)} = \int_{-a_i}^{a_i} [\sigma(x)u_y(x) + \tau(x)u_x(x)]\,\mathrm{d}x, \quad i = 1,2, \tag{2}$$

where $u_x(x)$ and $u_y(x)$ are the opening and sliding displacements of the microcrack, $\sigma(x)$ and $\tau(x)$ denote the normal and shear components of "pseudo-traction" [1] transferred to the microcrack surface. Summing over $\Pi^{(i)}$ $(i = 1,2)$ before and after the microcrack linkage, the difference in potential energy release $\Delta \Pi$ is obtained. The energy ratio R can then be found from Eq. (1).

4. Global softening

Microcrack connection in each linking step softens the material. The step-by-step global softening can be estimated by considering all possible microcracks linkages. The effective moduli prior to any microcrack linkage are given by [2]

$$E_0 = \frac{E_{\mathrm{matrix}}}{1 + 2\pi\rho}, \quad \nu_0 = \frac{\nu_{\mathrm{matrix}}}{1 + 2\pi\rho}, \tag{3}$$

where E_{matrix} and ν_{matrix} denote, respectively, the Young modulus and the Poisson ratio of the brittle matrix. The microcrack density is $\rho = (1/A) \times \sum_{i=1}^{N} a_i^2$ with A being the area of the configuration while N is the number of microcracks in A and a_i the half-length of the ith microcrack.

Fig. 2 shows two microcracks before and after coalescence. If they could connect for a ligament size c_1, then all neighboring microcracks with ligament size $c < c_1$ will connect. The percentage of the connected ligaments is given by $\eta(c_1) = \int_0^{c_1} p_0(c)\,\mathrm{d}c$.

After the microcracks link together, distributions of the microcrack half-lengths and ligament sizes should be adjusted to

$$\begin{aligned} f(a;c_1) &= \frac{1-2\eta(c_1)}{1-\eta(c_1)}\delta(a-a_0) + \frac{p_0(2a-4a_0)}{1-\eta(c_1)} \\ &\quad \times [H(2a-4a_0) - H(2a-4a_0-c_1)], \\ p(c;c_1) &= \frac{p_0(c)}{1-\eta(c_1)} H(c-c_1), \end{aligned} \tag{4}$$

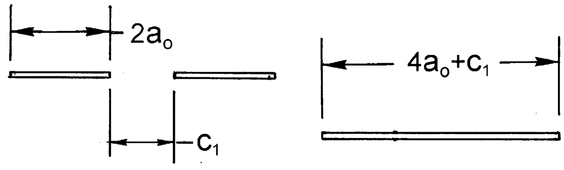

Fig. 2. Schematics of the initial microcrack linkage.

where $H(\cdot)$ denotes the Heaviside step function. The expected ligament size can be evaluated as

$$\bar{c}(c_1) = \int_{c_1}^{\infty} cp(c; c_1)\,\mathrm{d}c. \tag{5}$$

The microcrack density and the effective moduli can be modified to

$$\rho(c_1) = \frac{N}{A}\left\{[1 - 2\eta(c_1)]a_0^2 + \eta(c_1) \times \left[2a_0 + \frac{1}{2}\int_0^{c_1} cp_0(c)\,\mathrm{d}c\right]^2\right\}, \tag{6}$$

$$E(c_1) = \frac{E_{\mathrm{matrix}}}{1 + 2\pi\rho(c_1)}, \quad v(c_1) = \frac{v_{\mathrm{matrix}}}{1 + 2\pi\rho(c_1)}.$$

All the above parameters are labeled by the specific ligament size c_1. Integrating over all possible c_1, the following statistical estimate is obtained:

$$\langle y \rangle = \int_0^{\infty} y(c_1)p_0(c_1)\,\mathrm{d}c_1. \tag{7}$$

Symbol y may stand for η, p, f, $\bar{c}$, ρ, v or E.

The step-by-step microcrack linkages may be traced to evaluate the gradual softening effect. The result can be used in the energy ratio calculation to introduce the stress relaxation in each linking step.

5. Step-by-step R curves

Dimensional analysis dictates that R curves depend only on $K_{\mathrm{IC}}^2/(E_{\mathrm{matrix}}\gamma_s)$, Δ/Δ_0 and v_{matrix}, with $\Delta_0 = HK_{\mathrm{IC}}/(E_{\mathrm{matrix}}\sqrt{\pi a_0})$ being the critical displacement loading for a single crack of length $2a_0$. The values of $v_{\mathrm{matrix}} = 0.3$ and $K_{\mathrm{IC}}^2/E_{\mathrm{matrix}} = 1.950\gamma_s$ are used in the subsequent calculations. Fig. 4 depicts the step-by-step R versus c/a_0

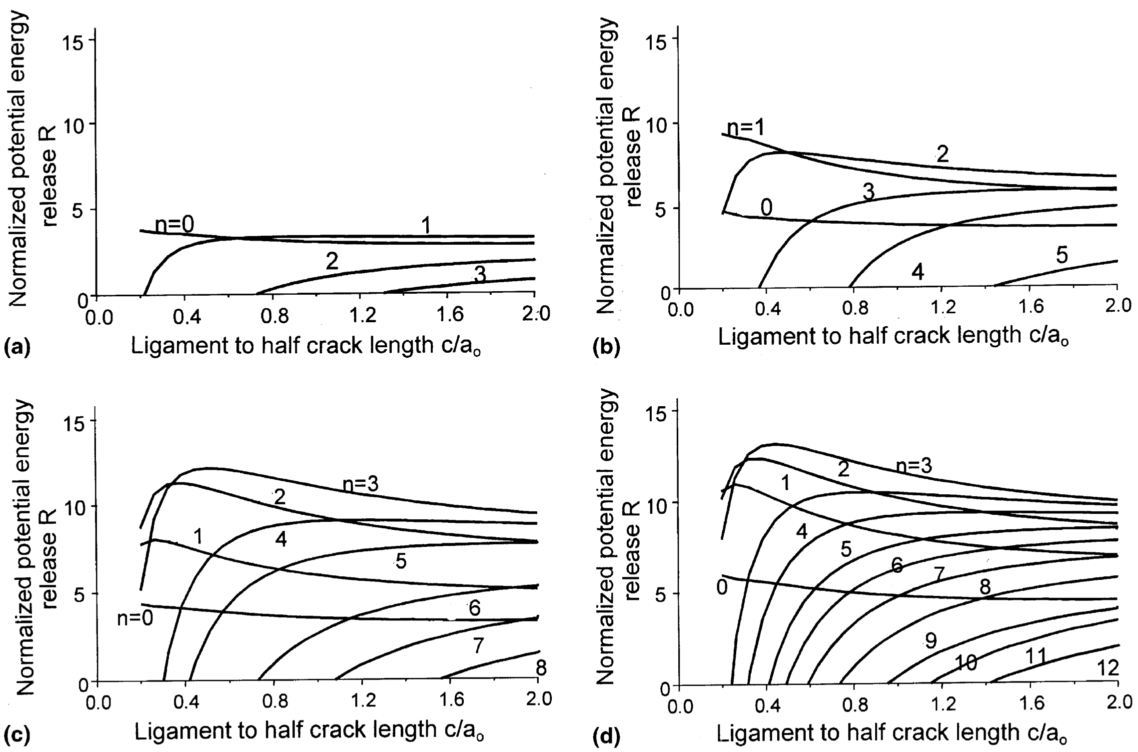

Fig. 3. Step-by-step R–c/a_0 curves under different normalized displacement loading: (a) $\Delta/\Delta_0 = 0.67$; (b) $\Delta/\Delta_0 = 0.71$; (c) $\Delta/\Delta_0 = 0.76$; (d) $\Delta/\Delta_0 = 0.82$.

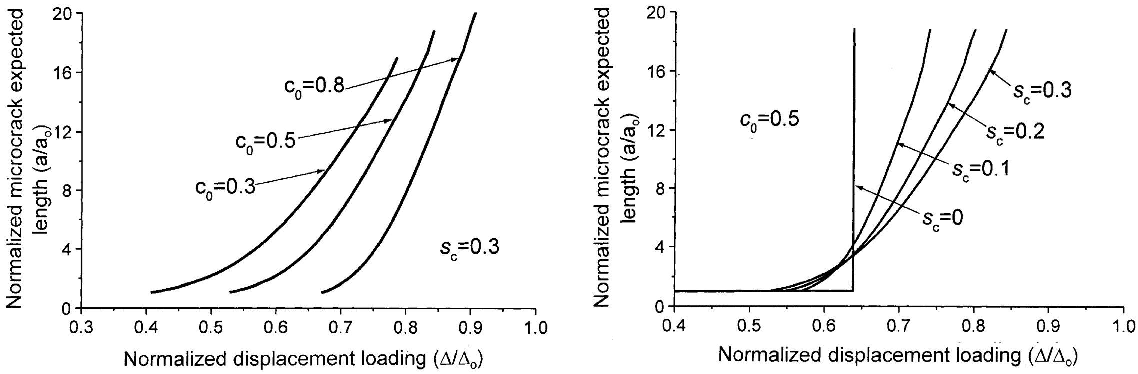

Fig. 4. Variation of the expected crack length under different displacement loading.

curves under different linking steps n. Various graphs correspond to different Δ/Δ_0 values. These curves are calculated under an expected ligament size c_0 of 0.5 and a standard deviation s_c of 0.3. Similar trends are found in all the graphs, i.e., the R level increases in the first few linking steps, then decreases by further linkages, and eventually falls to zero for a large range of c. The final stabilization and the expected length can thus be determined.

6. Expected length of coalescing microcracks

For the prescribed displacement and fixed microcrack geometry, one finds that the length of the longest connected crack corresponds to the one of vanishing crack driving force. The average of such lengths for statistically assigned microcrack configurations defines the expected length of coalescing microcracks. Four graphs in Fig. 3 indicate that the n value for the final stabilization, and consequently the expected length, increase with Δ/Δ_0. Fig. 4 plots the variation of expected crack length under different displacement loading. The curves in Fig. 4(a) are calculated for expected ligament sizes c_0 of 0.3, 0.5, 0.8 and standard variation s_c of 0.3. The plots show that the expected crack length increases as the displacement loading or the microcrack density increases. Furthermore, the expected crack length increases as the expected ligament size decreases. Curves in Fig. 4(b) are plotted for different s_c of 0.1, 0.2, 0.3 and a fixed c_0 value of 0.5. Aside from the similar trends as in Fig. 4(a), it is found that the expected length decreases as the standard deviation of the ligament size increases. Namely, the randomness of the microcrack distribution stabilizes the microcrack coalescence. The vertical curve of $s_c = 0$ corresponds to periodical microcracks and represents the most critical case of simultaneous fracture.

Fig. 5 shows the size effect of microcrack coalescence. Different curves correspond to microcrack numbers of 200, 500 and 1000 in the configuration, which is proportional to the specimen size. They are calculated under $c_0 = 0.3$ and $s_c = 0.3$. Given the amount of Δ/Δ_0, the specimen of a larger size would have a larger expected crack length. In a large specimen, short ligaments are

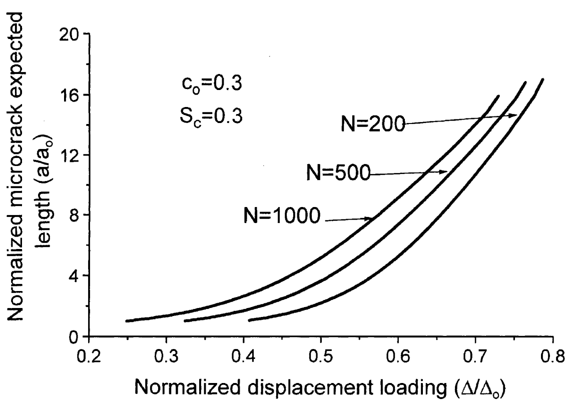

Fig. 5. Size effect of microcrack coalescence under displacement loading.

likely to appear and initial microcrack linkages are easier to proceed.

Acknowledgements

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