Coalescence of microcracks in brittle materials

LI Teng and YANG Wei

Department of Engineering Mechanics,

Tsinghua University, Beijing 100084, CHINA

Abstract

The fracture process of brittle materials with randomly oriented microcracks critically depends on the strong interaction among microcracks and the coalescence path leading to a fatal crack. In this paper, a model based on the coalescence process for planar orientated microcracks is presented. A energy ratio is defined between the total work dissipation and the new crack surface energy in one coalescence step, which is a token of microcrack propagation driving force in the energetic sense. The ratio is plotted against different configuration parameters for collinear and wavy microcrack arrays. Different microcrack coalescing modes are discussed, along with an estimate for the probability of microcrack coalescence dominated by the first linkage.

KEY WORDS: microcrack, coalescence, energy ratio, probability, fracture mode

Introduction

Randomly distributed microcracks impose a basic meso-damage configuration in brittle materials, which range from ceramics, concrete, to rocks. Previous investigation indicated that the strength of a brittle solid containing collinear microcracks depends not only on the average density of microcracks, but also on the fluctuation of microcracks, and the size of the specimen[1],[2]. For generally oriented microcracks, a similar understanding on the fracture process depends on the strong interaction among randomly oriented microcracks and the coalescence path leading to a fatal crack (Fig.1).

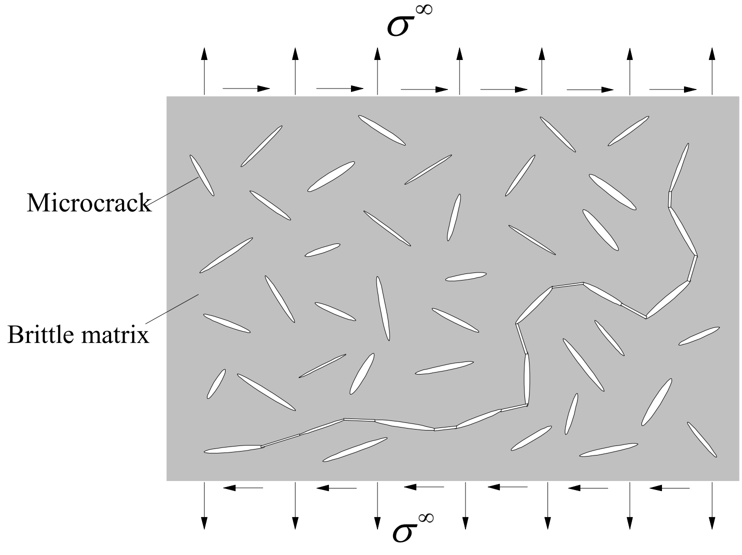


Fig 1 Schematic of randomly distributed microcracks and a coalescence path

In this paper, a model based on the coalescence process for planar orientated microcracks is presented. Attention is focused on the least energy linking path and the largest linking obstacle. Two crucial parameters, the total coalescence energy and the maximum energy barrier to connect the neighbouring microcracks, emerge during the analysis. The first parameter quantifies the global energetics for the coalescence, it correlates with the accumulated length of newly separated ligaments. The latter describes the controlling kinetics for the coalescence process, it focuses on each coalescence step determined by the geometry of microcracks and ligaments, accounting the effect of orientated microcracks.

The two parameters form a physical basis for the strength and the toughness of a brittle material containing microcracks. By considering all the possible coalescing processes of the given configuration and the maximum energy thresholds of each fracture path, the process from microcrack propagation to ultimate fracture can be statistically predicted. Further studies will link the microcracks distribution parameters of the configuration with the material strength and fracture toughness, which could improve the fracture prediction of brittle materials and optimize their mesoscopic structures.

Representative configuration

Assuming a self-similar coalescing process of microcracks. A representative configuration consists of the linkage of two neighbouring microcracks, as shown in Fig. 2.

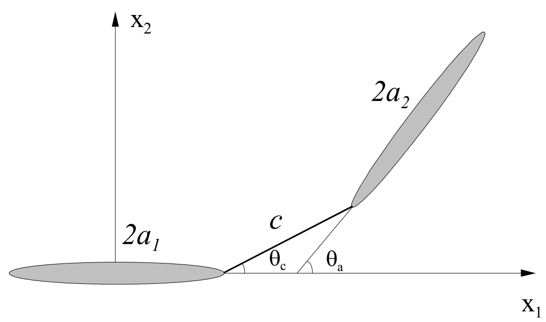


Fig. 2 Schematic of the representative configuration

Normalized configuration parameters are:

$$\left(\frac{a_1}{a_0}, \frac{a_2}{a_0}, \frac{c}{a_0}, \theta_a, \theta_c\right)$$

where a_1 and a_2 are the half-lengths of microcracks 1 and 2, c is ligament size, θ_c is the angle between microcrack 2 and the ligament, θ_a is the angle of two microcracks, and a_0 is the expected half-length of the randomly distributed microcracks, which is defined by $a_0 = \int_0^\infty aP(a)da$ ($P(a)$ is the density function of crack half-length).

The cracks can be simulated by continuous distribution of dislocations[4],[5]. The dislocation density are given by $D(x) = D_I(x) + iD_{II}(x)$, where the subscripts *I*, *II* label the fracture modes. One can expand the density function in terms of the Chebyshev polynomials of the first kind[3]. The stress and strain distributions can be expressed by the Chebyshev coefficients to be solved. The expected solution accuracy can be guaranteed by taking enough terms of Chebyshev polynomials.

Energy Ratio

The coalescence of the representative configuration is dictated by a energy ratio defined as:

$$R = \frac{\Delta\Pi}{2c\gamma_s} \tag{1}$$

where $\Delta\Pi$ denotes the release of potential energy due to the linkage of two neighbouring cracks, γ_s denotes the surface tension of the brittle matrix. Accordingly, R represents the ratio between the released potential energy and the energy dissipation along the linking surface during the coalescing step. The larger of R, the larger of the driving force, and the larger of the coalescence probability. With this understanding, one can define the fracture process as a sequence of coalescing steps with superior R values.

We denote $\sigma(x), \tau(x)$ as the normal and shear tractions transferred to the microcrack surface, whose opening and sliding displacements can be calculated by the scheme outlined in the preceding section. Then the potential energy contributed from the ith microcrack is given by:

$$\Pi^{(i)} = \int_{-a_i}^{a_i} (\sigma(x)u_y(x) + \tau(x)u_x(x))dx \quad i = 1,2 \tag{2}$$

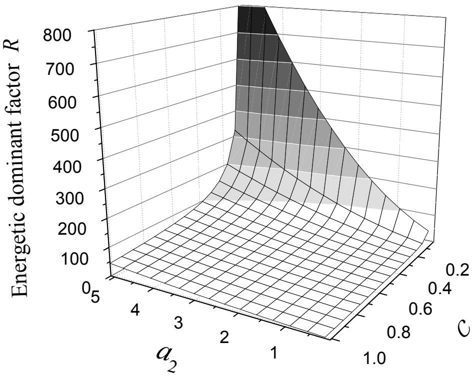
Summing over $\Pi^{(i)}$ before and after the microcrack linkage, one can obtain the potential energy release $\Delta\Pi$, then lead to the energy ratio R from (1).

Results

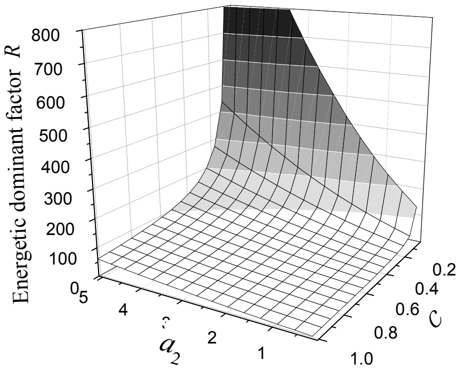
In this section, we give the numerical results of two special cases: collinear and wavy microcracks configurations.

Collinear microcracks

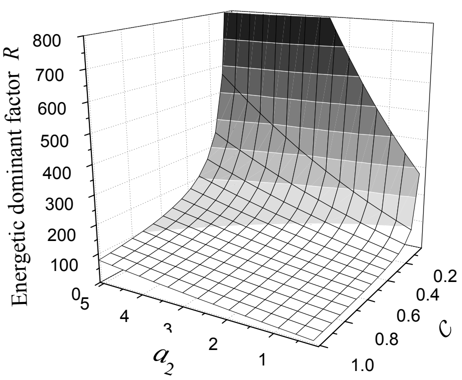
Collinear microcracks impose the most critical configuration under a prescribed microcrack density. Without loss of generality, one may fix the half-length of the first microcrack, and consider the variation of R with respect to a_2 and c. Various graphs in Fig. 3 describe the R surfaces with respect to a_2 and c for fixed values of a_1 from one to five. Grid lines in each graph correspond to curves under fixed values of a_2 or c. These graphs indicate that R increases as a_2 increases or as c decreases. The climb up of R becomes more rapid near the end of small c. For a given combination of a_1 and a_2, each R - c curve has a minimum at c^* (as shown in Fig.4a). When $0 < c < c^*$, R increases as c decreases, and shoots to infinity while c tends to zero; when $c > c^*$, R increases slowly as c increases, and approximately approaches a linear relation for fairly large c. Figure 4b depicts the R - c curves for small ligament size. They will serve as the basic curves for the subsequent analysis.



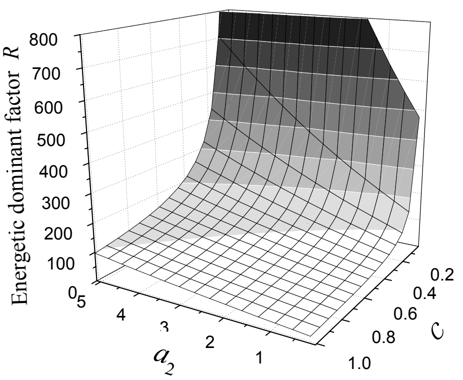
$a_1 = 1.0$



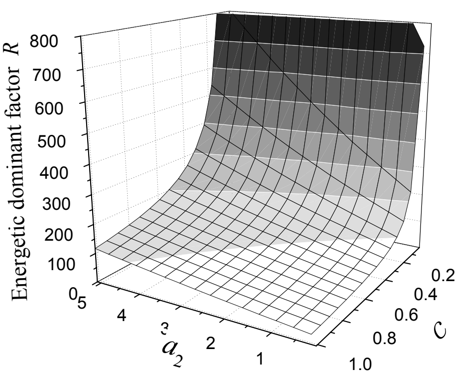
$a_1 = 2.0$



$a_1 = 3.0$

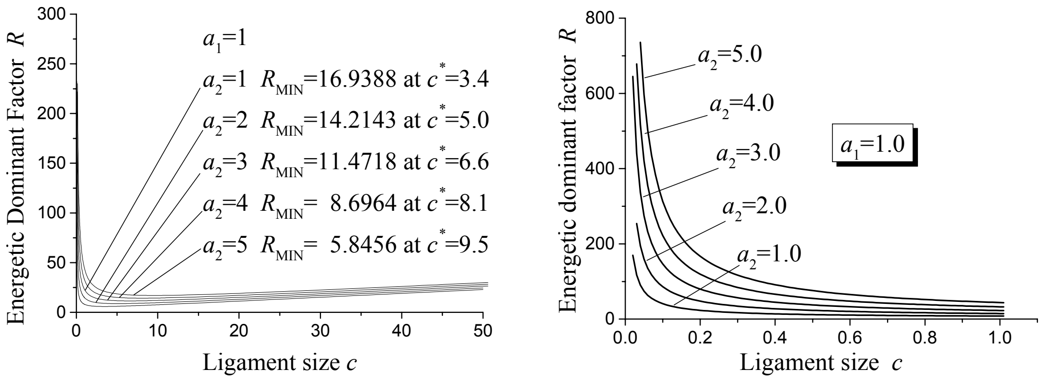


$a_1 = 4.0$



$a_1 = 5.0$

Fig. 3 R - (a_2, c) surfaces under different a_1 values

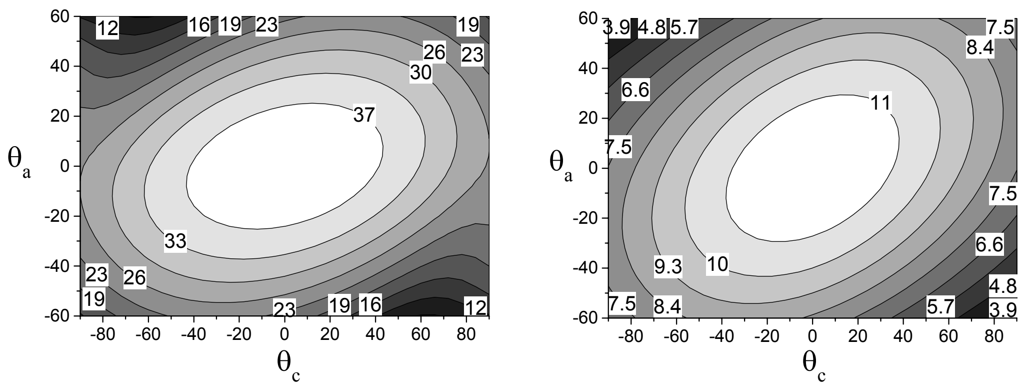


(*a*) (*b*)

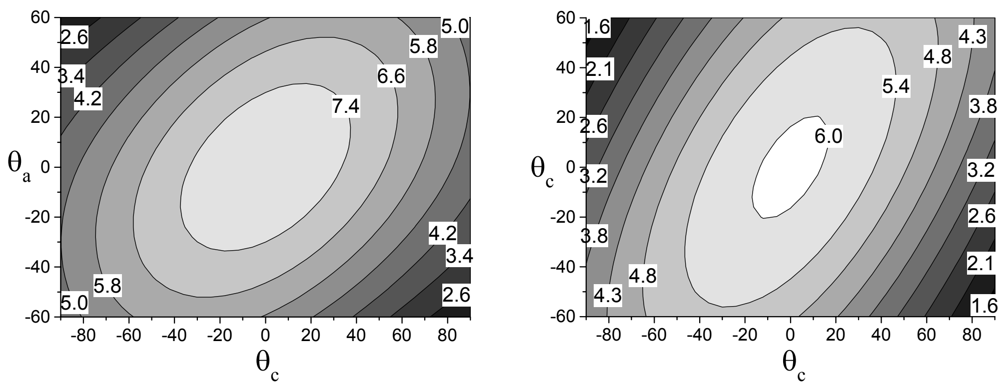
Fig. 4 R - c curves: (*a*) for large c range; (*b*) local zoom-out for small c

Wavy microcracks

Wavy microcrack array is a representative configuration for coalescing microcracks. We will illustrate the R contours under different combinations of microcrack lengths, ligament sizes, and oriented angles in this case hereafter.



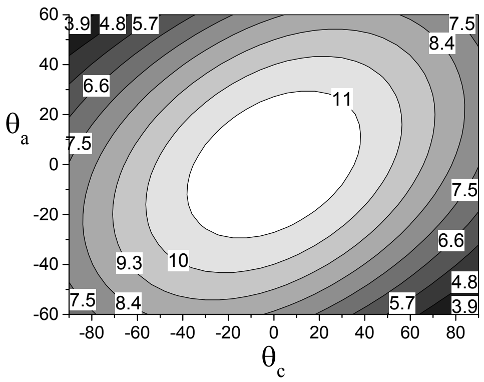
(a) c= 0.1 (b) c= 0.5



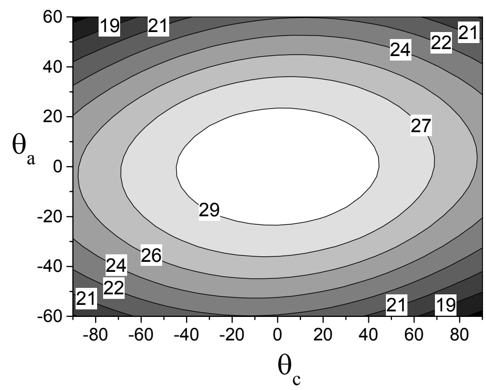
(c) c= 1.0 (d) c= 1.5

Fig. 5 Contours of $R-(\theta_\mathrm{a},\theta_\mathrm{c})$ surface with fixed a_1= a_2=1.0

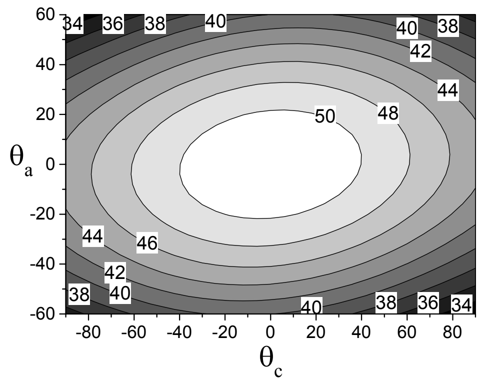
Figure 5(a)-(d) shows the contours of $R-(\theta_\mathrm{a},\theta_\mathrm{c})$ surface in the case of equal microcrack length a but different ligament sizes, where a=1 and c=0.1, 0.5, 1.0,1.5, respectively. All contours depict the similar decreasing trend of R with the increase of θ_a or θ_c, and R always reaches the maximum value at θ_a=0 and θ_c=0, which denotes the collinear configuration. For a given θ_a (or θ_c), the R-θ_c (or θ_a) curve reaches its peak point at a certain θ_c (or θ_a) value which has the same sign as θ_a (or θ_c). The larger of θ_a (or θ_c), the larger of the peak θ_c (or θ_a). This trend becomes more distinct in the case of larger *c*/*a* value. Comparing the four maps, one can find that the larger *a*/*c* value indicates the larger overall R level, thus stronger interaction, which is consistent with the collinear configuration.



(a) Initial configuration



(b) After the first linkage



(c) After the second linkage

Fig. 6 Contours of $R-(\theta_a,\theta_c)$ surface

Figure 6(a)-(c) show the contours of $R-(\theta_a,\theta_c)$ surface when *a*=1 and *c*=0.5. Different graphs correspond to different coalescing stages, that is, graph (a) denotes the initial configuration without any microcrack linkages; graph (b) denotes the configuration after the first linkage; while graph (c) the configuration after the second linkage. Tracking the variations of R with the coalescing steps, one finds that even the minimum R value in certain step exceeds the maximum R value of the former steps. For the case of equal microcrack length and ligament size, the microcracks connected by the first linkage will dominate and lead to the fatal collapse.

Discussion

We now discuss the relationship between the coalescing mode and the geometries of collinear microcrack configuration. We categorise the microcrack coalescence into two modes: the coalescence dominated by the first linkage and the coalescence dominated by subsequent linkages.

The first coalescing mode leads to catastrophic fracture without any precursors and relatively low loading capacity. The design objective of mesostructure in a brittle solid is to avoid that mode. Recall that microcrack coalescence occurs under strong interaction. Attention should be focused on the small ligament size range, say $c < c^*$. In that range, R increases as c decreases (as shown in Fig. 4b).

Consider that all microcracks have the same half-length of $a_0 = 1.0$ and are separated by ligaments whose sizes are described by a normal distribution $p(c)$. By tracing the coalescing process, the variation of R with respect to the ligament size c can be shown in Fig. 7.

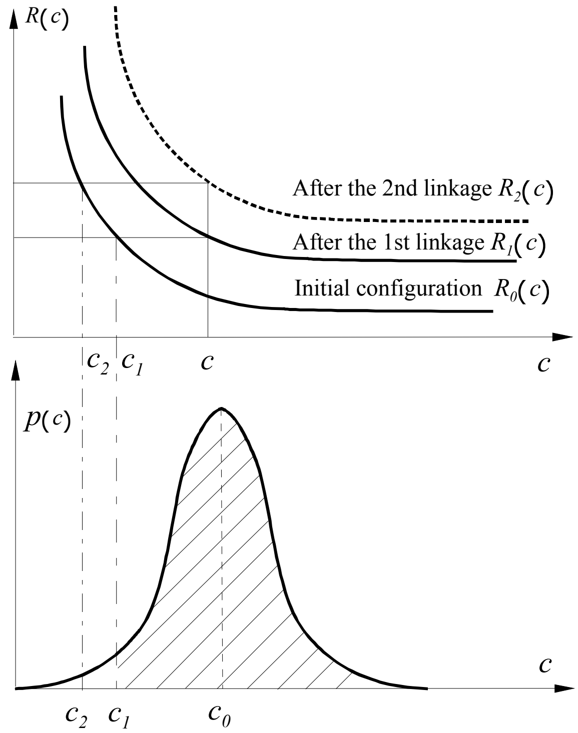


Fig. 7 Critical linkage in coalescing microcracks. Top graph shows R - c curves of different linkages, bottom graph depicts probability distribution of ligament size c

Figure 7 elucidates the concept of the critical linkage in statistically coalescing microcracks. The top graph plots energy ratios for the initial configuration ($R_0(c)$ curve), and those after the first linkage ($R_1(c)$ curve) and the second linkage ($R_2(c)$ curve). The bottom graph depicts $p(c)$, the normal probability distribution of ligament size with an expected value of c_0. For a given ligament size c, one can find an energy ratio $R_i(c)$ with a microcrack that has connected i times, $i = 1,2,\cdots$. Dictated by the same energy ratio, the linkage between two unconnected microcracks would require a shorter ligament size $c_i = R_0^{-1}(R_i(c))$, and consequently have less chance than the extension of the connected ones. The probability of microcrack coalescence dominated by the first linkage can be calculated by

$$P = \prod_{i=1}^{n_{fatal}} P_i \tag{3}$$

where n_{fatal} denotes the number of linkages to form a fatal crack, determined by the remote loading and matrix fracture toughness. The symbol P_i in (3) denotes the probability for the further extension of a microcrack that has already connected i times, and is calculated by

$$P_i = \int_0^\infty p(c) \int_{c_i(c)}^\infty p(c')\mathrm{d}c'\,\mathrm{d}c \tag{4}$$

Given a certain microcrack distribution and geometrical parameters, one can quantitatively determine the probability of microcrack coalescence dominated by the first linkage. Figure 8 plots the variation of P under different expected ligament sizes and standard deviations. The curves are calculated under $a_0 = 1.0$ and c_o = 0.3, 0.5, 0.8, 1.0. The plots depict that the probability of first linkage dominated fracture decreases as s increases, where s measures the randomness of the microcrack distribution. The case of $s = 0$ corresponds to periodical microcracks and is always dominated by the first linkage. A transition value s_{tran} of the standard deviation of ligament sizes can be defined. When $s < s_{tran}$, the microcrack coalescence is dominated by the first linkage, otherwise it is dominated by the subsequent linkages. As the expected ligament size increases, the transition value s_{tran} increases and the transition becomes smoother.

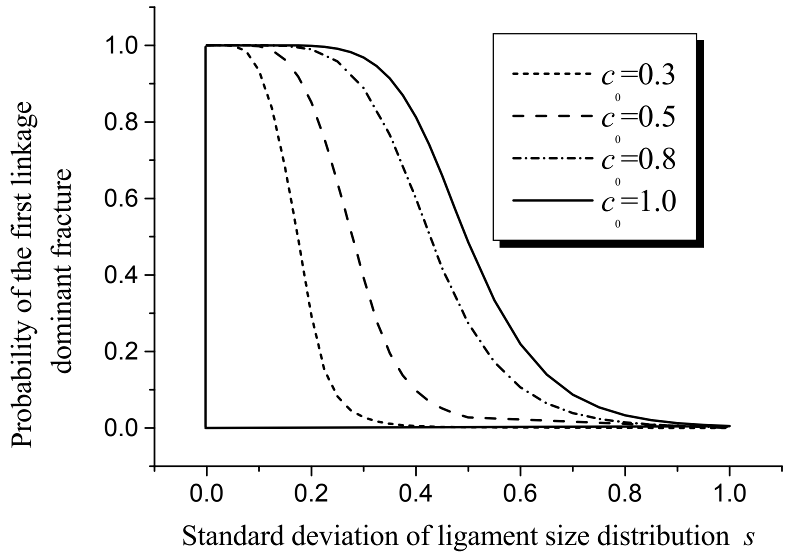


Fig. 8 Probability of microcrack coalescence dominated by the first linkage

Conclusion

1. The coalescence of microcracks is controlled by energy ratio R between the release of potential energy and the energy required to create the new crack surface.
2. The influences on R by various geometrical and statistical parameters are calculated for collinear and wavy microcrack configurations.
3. For wavy array consisting of microcracks of equal length and ligament size, the microcrack coalescence will be dominated by the first linkage, no matter what random orientions the microcracks have.

4. For collinear microcracks configuration, the first linkage dominated fracture becomes less probable as the standard deviation of ligament size increases. This quantitative relation can serve in the optimal design of material meso-structure.

Reference

[1] Su-lin ZHANG, Teng LI and Wei YANG (1998) *International Journal of Solids and Structures.* Vol. 35, No. 11, pp. 995-1008

[2] Wei YANG, Su-lin ZHANG and Teng LI (1997) *Proceedings of the Ninth International Conference on Fracture.* Vol. 4, pp. 1971-1978

[3] F. ERDOGAN, G.GUPTA (1971) *International Journal of Solids and Structures.* Vol. 7, pp. 39-61

[4] Xueli HAN, Tsuchiang WANG (1996) *International Journal of Fracture*. Vol. 76, R69-R72

[5] R. FREIJ-AYOUB, A. V. DYSKIN and A. N. GALYBIN (1997) *International Journal of Fracture*. Vol. 86, L57-L62