LABORATORY 8 – Passive Filter Designs

Lab Goals

For this lab you will design, scale, construct, and test passive filter circuits. You will compare the frequency performance of two different filters.

Definitions

Band Notch Filter - A circuit that rejects a range of frequency signals to a circuit but allows low and high frequency signals to pass through with a fairly constant gain.

Band Pass Filter - A circuit that allows a range of frequency signals to pass through a circuit but attenuates low and high frequency signals.

Butterworth Filter - A type of filter that is "maximally flat", i.e. as many derivatives of the magnitude of the transfer function as possible are equal to zero at some specified frequency.

Corner Frequency - The frequency where the power in the output signal is 50% of the output power in the passband. This is also known as the 3dB frequency and the breakpoint frequency.

Filter Circuit - A circuit that conditions an input signal, usually by allowing only a range of frequencies to pass through the circuit to the output. The purpose of a filter is often to separate a desired signal from undesired signals (e.g. noise).

High Pass Filter - A circuit that allows high frequency signals to pass through a circuit but attenuates low frequency signals.

Low Pass Filter - A circuit that allows low frequency signals to pass through a circuit either unattenuated or with a fairly constant gain (or loss) but attenuates high frequency signals.

Normalized Filter Circuit - A circuit whose parameters are adjusted so that the corner frequency is 1 radian/sec and the output impedance is 1 Ω.

Passband - The range of frequencies that passes through a filter relatively unattenuated (or at least fairly uniformly).

Passive Filter – filters constructed without op-amps. Typically have an insertion loss of 6 dB.

Roll Off - The rate (in dB/decade) that the power in a signal is attenuated outside of the passband as a function of frequency.

Circuit Analysis

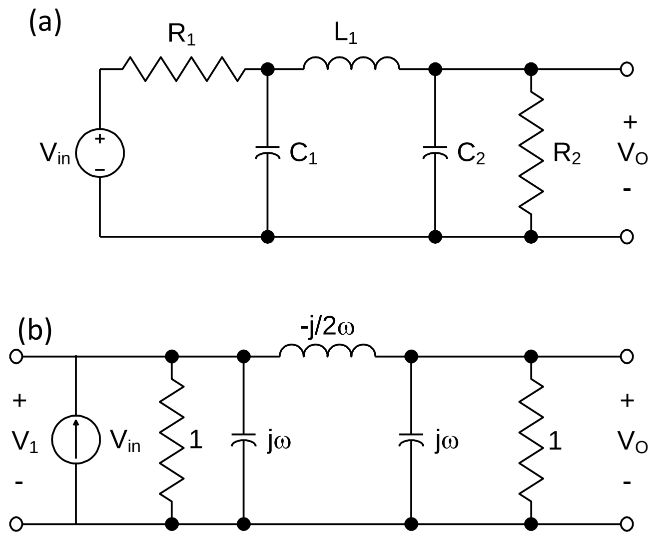


Figure 9.1 A normalized passive Butterworth low pass filter.

In this section we will analyze a passive, 3rd order, low-pass filter. It is called a passive circuit because it has only passive components (R, L, and C) as is shown in Fig. 9.1a. At low frequencies, capacitors act as open circuits and inductors act as short circuits, so the circuit is a simple resistive divider at low frequencies with

$$\hat{V}_0 / \hat{V}_{in} = \frac{R_2}{R_2 + R_1}.$$

At high frequencies, inductors have high impedance and capacitors have low impedance. Because the capacitors in Fig. 9.1a are in parallel either with the input or the output and the inductor is in series (sort of) with the output, all three components tend to reduce the output signal at high frequency. Therefore, this circuit is called a 3rd-order low pass filter. The details of the frequency response depend on the particular values of the components.

We will use general nodal analysis to model a normalized filter circuit where we take the values in Fig. 9.1a to be $C_1 = C_2 = 1\text{F},\ R_1 = R_2 = 1\Omega,$ and $L_1 = 2\,\text{H}$. The resultant admittances are indicated in Fig. 9.1b, where we have also transformed the nonideal voltage source into a nonideal current source. There are three nodes, so we take the lowest node to be ground and find the nodal equations in matrix form to be:

$$\begin{pmatrix} 1 + j\omega - {}^{j}\!/_{2\omega} & {}^{j}\!/_{2\omega} \\ {}^{j}\!/_{2\omega} & 1 + j\omega - {}^{j}\!/_{2\omega} \end{pmatrix} \begin{pmatrix} \hat{V}_1 \\ \hat{V}_{out} \end{pmatrix} = \begin{pmatrix} \hat{V}_{in} \\ 0 \end{pmatrix}$$

which can be inverted to find

$$\begin{pmatrix}\hat{V}_1\\ \hat{V}_{out}\end{pmatrix} = \frac{\begin{pmatrix}1+j\omega-j/2\omega & -j/2\omega\\ -j/2\omega & 1+j\omega-j/2\omega\end{pmatrix}\begin{pmatrix}\hat{V}_{in}\\ 0\end{pmatrix}}{\left(1+j\omega-j/2\omega\right)^2-\left(j/2\omega\right)^2},$$

or

$$\hat{V}_{out} = \frac{\hat{V}_{in}/2j\omega}{(1+j\omega-j/2\omega)^2-(j/2\omega)^2},$$

or simply

$$\hat{V}_{out}/\hat{V}_{in} = \frac{1/2}{1+2j\omega+2(j\omega)^2+(j\omega)^3}.$$

The magnitude of the transfer function is

$$\left|\hat{V}_{out}/\hat{V}_{in}\right| = \frac{1/2}{\sqrt{\left(1-2\omega^2\right)^2+\omega^2(2-\omega^2)^2}} = \frac{1/2}{\sqrt{1+\omega^6}}$$

after a little algebra. This filter is known as a (3rd-order) Butterworth filter. It is also called a "maximally-flat" low-pass filter because the first $2n-1$ $(n=3)$ derivatives of the transfer function are zero at $\omega=0$. A sketch of the transfer function magnitude, or gain, is shown in Fig. 9.2 as a function of frequency. The maximum voltage gain is ½ at low frequencies and the corresponding power gain is ¼ since for a resistive load, the power is proportional to the voltage squared. The frequency range where the gain is nearly flat is called the pass band. The corner frequency where the gain has decreased by the square root of two is $\omega = 1\ \mathrm{rad/sec}$. Above this frequency the output voltage "falls off" from the maximum value by three orders of magnitude for every decade increase in frequency.

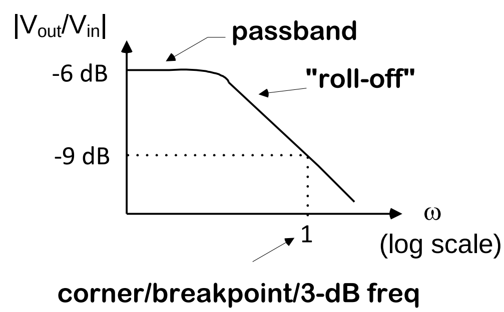


Figure 9.2 Frequency response of a low pass filter.

It is often useful to take the logarithm of the transfer function:

$$\left|\frac{\hat{V}_{out}}{\hat{V}_{in}}\right|(\mathrm{dB}) = 20\log_{10}\left|\hat{V}_{out}/\hat{V}_{in}\right|$$
$$= 20\log_{10}(1/2) - 20\log_{10}\sqrt{1+\omega^6}$$
$$\approx -6 - 10\log_{10}\left(1+\omega^6\right)$$

For low frequencies

$$\left|\hat{V}_{out}/\hat{V}_{in}\right| = -6 \text{ dB.}$$

This value is called the insertion loss. For high frequencies $(\omega >> 1)$,

$$\left|\hat{V}_{out}/\hat{V}_{in}\right| \approx -6 - 60\log_{10}(\omega) \text{ dB}$$

Thus, we say the roll-off is approximately 60dB/decade. The roll-off of a low pass filter is always 20dB/decade times the order of the filter (3 in this case).

Typically, a filter that has a corner frequency of 1 rad/sec and is used with a load impedance of $1\,\Omega$ is not very interesting. But the process of filter synthesis via frequency and impedance denormalization can be used to transform this design into one with a desirable corner frequency and load impedance. This is nice, because every time you need to design a filter for a new frequency or impedance, you don't need to start from scratch. If the required load impedance is R_L and the required corner frequency is ω_0, simply take any values of R, L, and C that you find in the normalized circuit and replace them with $R', L',$ and C', respectively, according to the following formulas:

$$R' = R * R_L$$
$$L' = L * R_L / \omega_0$$
$$C' = C / (R_L * \omega_0)$$

As a concrete example, say we want to have a load impedance of $51\,\Omega$ and a corner frequency of 3.410 kHz for our 3rd-order low pass filter, we would replace both resistances with $51\,\Omega$ (the resistor on the right is the load resistor, by the way). The 2H inductor would be replaced by

$$L' = 2 * 51/(2\pi \times 3.45 \times 10^3) = 4.7\text{mH}$$

and the capacitors would be

$$C' = 1/(51 * 2\pi \times 3.45 \times 10^3) = 0.9\ \mu\text{F}.$$

That's all there is to scaling!

Let's say that we wanted to design a normalized 3rd-order high-pass passive filter, given that we are aware of the 3rd-order low-pass design analyzed previously. To convert the low-pass design to a high-pass design, we simply need to replace the inductor with a capacitor and the capacitors with inductors. The component values are reciprocal so that the impedances are identical when ω=1. For example, if an inductor with inductance L (H) is to be replaced with a capacitor, the value of the capacitance (in Farads) is 1/L.

To convert a passive low-pass filter to a band-pass filter, one must replace all capacitors with parallel LC combinations. Inductors must be replaced by series LC combinations. To convert a low-pass filter to a band-notch filter, the opposite must be done: inductors are replaced with parallel LC combinations and capacitors are replaced by series LC combinations. The four types of normalized 3rd-order Butterworth filters are summarized in Fig. 9.3. There is an additional degree of freedom in the band-pass and band-notch filters that can be used to adjust the Q of the frequency variation, but that discussion is left for other texts. Here it suffices to say the product LC must be chosen to give the proper center frequency ($\omega_c = 1/\sqrt{LC}$), but the ratio L/C is free to be adjusted to modify the bandwidth of the resonance.

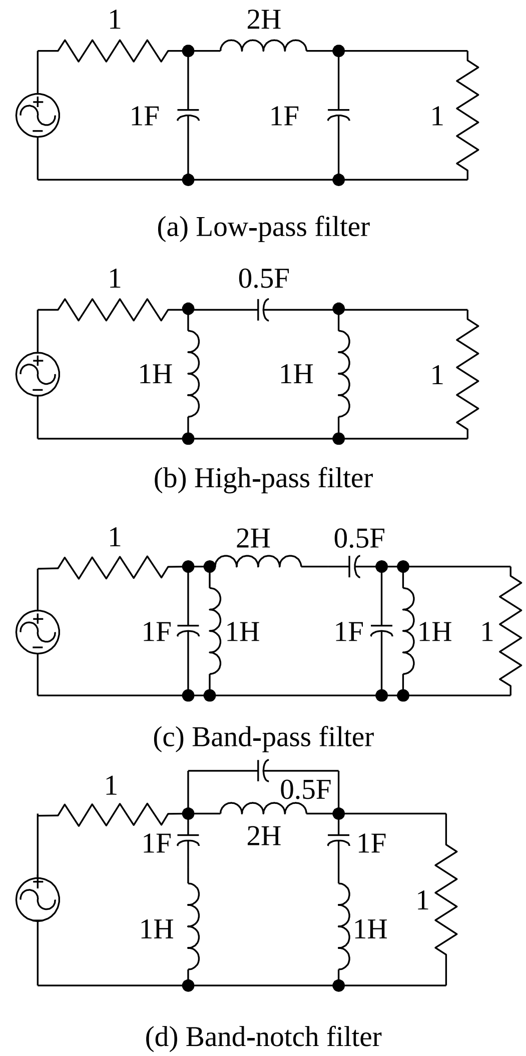
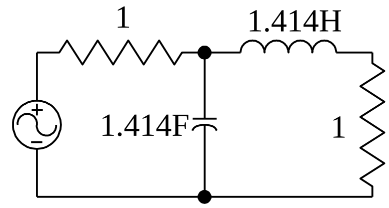


Figure 9.3 Standard 3rd order passive Butterworth filters.

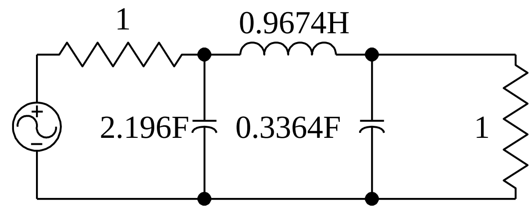
A number of other normalized low-pass filters are shown in Fig. 9.4. The filter descriptions and orders are given in the figure. The properties of the Butterworth filters are similar to the 3rd order circuit that we discussed – the attenuation outside the passband simply varies with the order of the filter. The Tchebychev filters are designed to minimize the ripple in the pass-band. There are many other types of filters, whose properties are discussed in the literature.

G. Helpful Hints

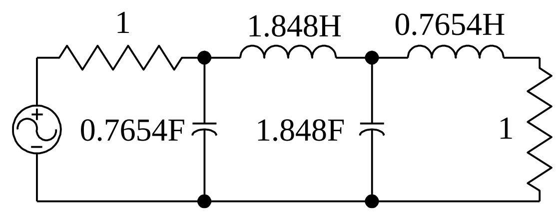
1. In this lab we have a very limited selection of inductors. Consider using parallel or series combinations of inductors to get the necessary values.
2. If necessary, adjust the frequency of your design slightly (no more than 10%) to reflect the inductance values that are available to you.
3. When simulating the inductor in PSpice, use a series resistor to account for inductor resistance.



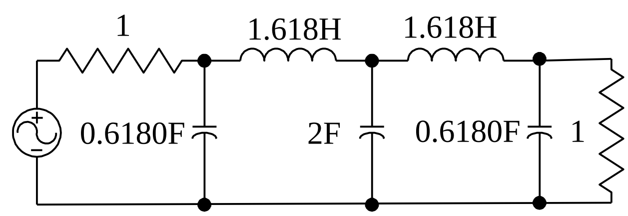
(a) 2nd order Butterworth filter.



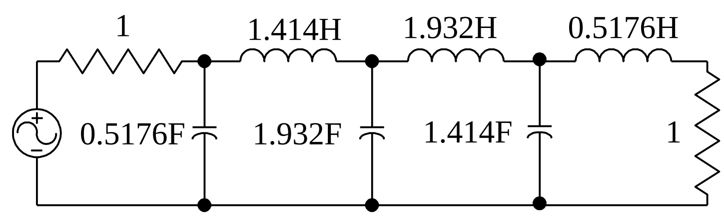
(b) 3rd order Gaussian filter.



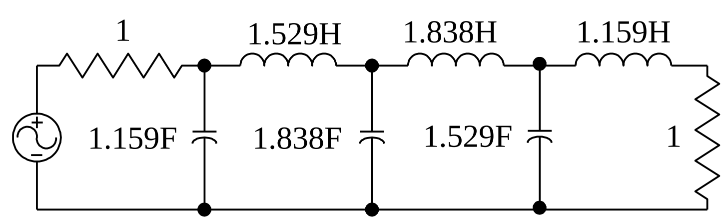
(c) 4th order Butterworth filter.



(d) 5th order Butterworth filter.



(e) 6th order Butterworth filter.



(f) 6th order Tchebychev filter.

Figure 9.4 Other normalized low-pass passive filters.

Laboratory 8 Description - Passive Filter Designs

Objective:

To design, build, analyze, compare and contrast the performance of two filter circuits.

Pre-lab preparation

Part I – First Filter

1. Design a passive, low-pass, second order, Butterworth filter with a corner frequency of about 10 kHz and a load resistance of about 51 Ω. Draw the circuit diagram.
2. Use PSpice to simulate the circuit performance. Plot the output voltage over a suitable range of frequencies. Plot both the magnitude and phase of the output. Assume the input voltage is 2 V peak-peak.
3. Double the output resistance in the circuit and repeat the simulations.

Part II – Second Filter

4. Design a passive, high-pass, second order, Butterworth filter with a corner frequency of about 5 kHz and a load resistance of about 51 Ω. Draw the circuit diagram.
5. Draw the circuit diagram.
6. Use PSpice to simulate the circuit performance. Plot the output voltage over a suitable range of frequencies. Plot both the magnitude and phase of the output. Assume the input voltage is 10V peak-peak.
7. Double the output resistance in the circuit and repeat the simulations.

Part III –Filter Combination

8. In PSpice, connect the output of the first filter to the input of the second filter and simulate the circuit performance. Plot the output voltage (of the second filter) over a suitable range of frequencies. Plot both the magnitude and phase of the output. Assume the input voltage (to the first filter) is 2 V peak-peak.

Experimental procedure:

Part I – First Filter

1. Construct the first filter circuit. Set the input voltage to 2 V peak-peak.

Measure the relative magnitude and phase of the output voltage from 50 Hz to 500 kHz. Find the 3 dB point and plot the input and output voltages.

2. Double the load resistance and repeat the previous measurement.

Part II – Second Filter

3. Construct the second filter circuit. Set the input voltage to 2 V peak-peak.
4. Measure the relative magnitude and phase of the output voltage from 50 Hz to 500 kHz. Find the 3 dB point and plot the input and output voltages.
5. Double the load resistance and repeat the previous measurement.

Part III – Combined Filters

6. Connect the output of the first filter to the input of the second filter. For this new, combined filter, measure the relative magnitude and phase of the output voltage from 50 Hz to 500 kHz. Find the 3 dB points and plot the input and output voltages.

Post-lab analysis:

Generate a lab report "following" the sample report available in Appendix A. Mention any difficulties encountered during the lab. Describe any results that were unexpected and try to account for the origin of these results (i.e. explain what happened). In ADDITION, answer the following questions:

Part I – for the first filter:

1. Compare the measured performance with the simulation results and the analytic expectations for the filter with the matched load. Discuss both the relative amplitude and phase of the output qualitatively.
   1.1. What is the measured pass-band gain of the circuit?
   1.2. What is the corner / center frequency of the circuit?
   1.3. What is the measured roll-off (in dB/decade) far away from the 3 dB point of the circuit?
2. Compare the measured performance with the simulation results for the filter with the unmatched (doubled) load. Discuss both the relative amplitude and phase of the output qualitatively.
   2.1. What is the measured pass-band gain of the circuit?
   2.2. What is the corner / center frequency of the circuit?
   2.3. What is the measured roll-off (in dB/decade) far away from the 3 dB point of the circuit?

3. Plot the frequency response (**magnitude and phase**!!!) on a log-log graph for the frequency sweep mentioned in the experimental section above.

Part II – for the second filter:

4. Compare the measured performance with the simulation results and the analytic expectations for the filter with the matched load. Discuss both the relative amplitude and phase of the output qualitatively.
   4.1. What is the measured pass-band gain of the circuit?
   4.2. What is the corner / center frequency of the circuit?
   4.3. What is the measured roll-off (in dB/decade) far away from the 3 dB point of the circuit?
5. Compare the measured performance with the simulation results for the filter with the unmatched (doubled) load. Discuss both the relative amplitude and phase of the output qualitatively.
   5.1. What is the measured pass-band gain of the circuit?
   5.2. What is the corner / center frequency of the circuit?
   5.3. What is the measured roll-off (in dB/decade) far away from the 3 dB point of the circuit?
6. Plot the frequency response (**magnitude and phase**!!!) on a log-log graph for the frequency sweep mentioned in the experimental section above.

Part III – for the combined filter:

7. Compare the measured performance with the simulation results for the combination. Discuss both the relative amplitude and phase of the output qualitatively.
   7.1. What is the measured pass-band gain of the circuit?
   7.2. What are the corner frequencies of the circuit?
   7.3. What is the measured roll-off (in dB/decade) far away from the 3 dB point of the circuit?
8. Plot the frequency response (**magnitude and phase**!!!) on a log-log graph for the frequency sweep mentioned in the experimental section above.